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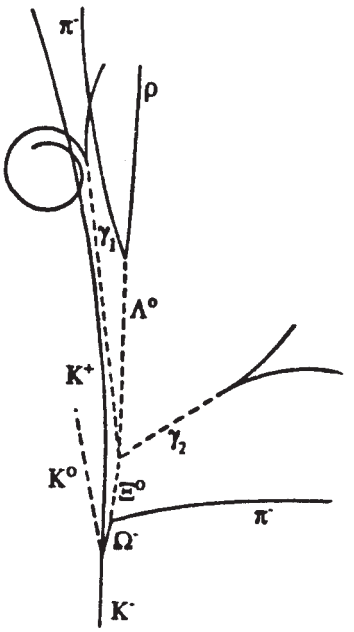

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VOLUME 37, NUMBER 2, APRIL 2014

HERMITIAN TRIPLE SYSTEMS ASSOCIATED WITH BILINEAR FORMS AND CERTAIN APPLICATIONS TO FIELD THEORY, 131
Noriaki Kamiya
Department of Mathematics, University of Aizu
Aizuwakamatsu, 965-8580, Japan
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Department of Natural Science, Faculty of Education
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Bunkyo-cho 1, Hirosaki, Aomori 036-8560, Japan

SOME CLARIFICATION AND CORRECTION OF RECENT CRITICISM OF EINSTEIN, 149
Jeremy Dunning-Davies
Department of Physics, University of Hull
Hull, HU6 7RX, England and
Institute for Basic Research, Palm Harbor, FL, USA

SEVERAL ROUTES TO DETERMINING ENTROPY GENERATION IN THE EARLY UNIVERSE/LINKS TO CMBR SPECTRA/RELIC NEUTRINO PRODUCTION, 155
Andrew Beckwith
Chongqing University, Department of Physics
Chongqing, PRC, 400044

FOUNDATIONS OF ISOMATHEMATICS, 163
Arun S. Muktibodh
Department of Mathematics
Mohota College of Science
Nagpur-440009, India

CONSEQUENCES OF SQUEEZING INITIALLY COHERENT (SEMI CLASSICAL) STATES WITH REGARDS TO FINDING IF THE COSMOLOGICAL CONSTANT IS OR IS NOT A 'VACUUM' FIELD, 179
Andrew Beckwith
Chongqing University, Department of Physics
Chongqing, PRC, 400044

EXTERIOR AND INTERIOR DYNAMIC ISO-SPHERE HOLOGRAPHIC RINGS WITH AN INVERSE ISO-DUALITY, 227

Nathan O. Schmidt
Department of Mathematics
Boise State University
1910 University Drive
Boise, ID 83725, USA

THE GALACTIC BLACK HOLE, 241

Kenneth Dalton
General Delivery
Bang Saphan Yai 77140
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HADRONIC JOURNAL 37, 131 - 147 (2014)

HERMITIAN TRIPLE SYSTEMS ASSOCIATED WITH BILINEAR FORMS AND CERTAIN APPLICATIONS TO FIELD THEORY

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Abstract

We define hermitian triple systems and give several examples induced from certain bilinear forms. We apply such a system to a field theory and obtain a Chern-Simons gauge theory.

Keywords: generalized Jordan triple systems; Lie algebras; hermitian operators; 3-algebra; Chern-Simons gauge theories; M-theory.

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1 Introduction

The field of nonassociative algebras is rich in algebraic structure. It provides an important common ground for various branches of mathematics and physics, not only for Jordan algebras, Lie algebras and triple systems, but also for applications to M-theory and field theory in physics. Thus we will discuss the concept of triple systems with a ternary product $< xyz >$ for a study of nonassociative algebras, in particular.

The structure theory of the generalized Jordan triple systems of second order of finite dimensional or called Kantor triple systems, which contains the concept of Jordan algebras and Jordan triple systems have been studied by author or other mathematicians ([1-13]).

On the other hand, it is well known ([14-24]) that the symmetric bounded domains are a one to one, correspondence to hermitian Jordan triple systems, for which a certain trace form is positive definite hermitian and that the concept of triple systems is useful to construct of Lie algebras or superalgebras.

Hence as a generalization of these concept, it seems that we are interesting to investigate for structure theory of hermitian generalized Jordan triple systems, in particular, the case of hermitian generalized Jordan triple systems of second order.

We shall concerned with algebras and triple systems which are infinite over a complex number field, unless otherwise specified.

In the point of view of string theory, the hermitian generalized Jordan triple systems are generalizations of the hermitian 3-algebras ([25-67]), which have been played crucial roles in M-theory. The field theories applied with the hermitian 3-algebras are the Chern-Simons gauge theories that describe effective actions of membranes in M-theory. Moreover, 3-algebra models of M-theory itself were proposed and have been studied in ([68-71]). Therefore, it is interesting to apply the hermitian generalized Jordan triple systems to the field theories.

We defined hermitian generalized Jordan triple systems and hermitian (ϵ, δ)-Freudenthal-Kantor triple system and proved structure theorems in ([72,73]). In this paper, we give several examples of the triple systems induced from certain bilinear forms. We apply such a system to a field

theory and obtain a Chern-Simons gauge theory. We also give some other examples without generalized Jordan triple systems or (ϵ, δ)-Freudenthal-Kantor triple systems and examples of nonassociative algebras given by our triple systems.

2 Definition and preamble

In this section, we give several definitions and the preamble in order to make present paper as self-contained as possible.

Definition. A triple systems U is said to be a *-generalized Jordan triple system of second order if relations (0)–(iv) satisfy;

0) U is a Banach space,

i) $[L(a,b), L(c,d)] = L(< abc >, d) - L(c, < bad >)$,

ii) $K(< abc >, d) + K(c, < abd >) + K(a, K(c,d)b) = 0$,

where $L(a,b)c =< abc >$and $K(a,b)c =< acb > - < bca >$,

iii) $< xyz >$ is **C**-linear operator on x, z and **C**-anti-linear operator on y,

iv) $< abc >$ continuous with respect to a norm $||\ ||$ that is, there exists $K > 0$ such that

$$|| < xxx > || \leq K||x||^3 \text{ for all } x \in U.$$

Furthermore, a *-generalized Jordan triple system of second order is said to be hermitian if it satisfies the following condition,

v) all operator $L(x,y)$ is a positive hermitian operator with a hermitian metric

$$(x,y) = trace\ L(x,y),$$

that is, $(L(x,y)z, w) = (z, L^*(x,y)w)$, and $(x,y) = \overline{(y,x)}$.

In particular, if a triple system U satisfies the conditions (o),(i),(iii),(iv) and (v), then it is said to be a hermitian generalized Jordan triple system.

Furthermore, as a generalization, it is said to be a (ε, δ)-Freudenthal-Kantor triple system if the following relations satisfy;

i)' $[L(a,b), L(c,d)] = L(< abc >, d) + \varepsilon L(c, < bad >)$,

ii)' $K(< abc >, d) + K(c, < abd >) + \delta K(a, K(c,d)b) = 0$,

where $K(a,b)c =< acb > -\delta < bca >$, $\varepsilon = \pm 1, \delta = \pm 1$.

For (ε, δ) Freudenthal-Kantor triple systems, its (v) is generalized as follows,

(v)' $(L(x,y)z,w) = -\varepsilon(z, L^*(x,y)w)$, and $(x,y) = -\varepsilon\overline{(y,x)}$.

Also, in particular, it is said to be a (ε, δ) Jordan triple system if $K(x,y) = 0$ for all $x, y \in U$.

We note that the concept of generalized Jordan triple system of second order coincides with that of (-1,1)-Freudenthal-Kantor triple system.

Example 1.1 *Let U be a J^*-triple (for the definition, to see [15]). Then, U is a hermitian generalized Jordan triple system of second order, because $K(a,b)c =< abc > - < cba >$ and $K(a,b)c \equiv 0$ (identically zero).*

Example 1.2. *Let $T^*_{n,n}$ be the space of diagonal matrix of $n \times n$ with element of the complex number. Then $T^*_{n,n}$ is a *-generalized Jordan triple system of second order, with respect to the product and the norm $||\ ||$*

$$< xyz >= x\bar{y}^T z + z\bar{y}^T x - \bar{y}x^T z$$

for all $x, y, z \in T^*_{n,n}$

$$||x|| = \max(|\lambda_i|, |\mu_i|)$$

where $x = \Sigma(\lambda_i e_i + \mu_i\sqrt{-1}e_i)$, and e_i are matrix unit element of $T^*_{n,n}$ and x^T is the transpose of x.

Remark We note that if $(U, < xyz >)$ is a (ε, δ) Freudenthal-Kantor triple system with an endomorphism P of U s.t. $P^2 = Id$, and $P < xyz >=< PxPyPz >$, then new triple product defined by $\{xyz\} =< xPyz >$ is a (ε, δ)Freudenthal-Kantor triple system.

Also, we have an example of P as follows:

For any $x = (x_1, \cdot\cdot, x_n) \in U$, we define

$$P(x) = \bar{x} := (\bar{x}_1, \cdot\cdot, \bar{x}_n). \in U,$$

where $dim_{\mathbf{C}}U = n$ and all $\bar{x}_i$ denotes the conjugation of the complex number $\mathbf{C}$, then we have

$$P < xyz >=< PxPyPz > \ and\ P^2 = Id.$$

Let U be a *-generalized Jordan triple system. Then we may define the notation of tripotent and bitripotent as follows.

Definition. It is said to be a tripotent of U if

$$< ccc >= c,\ c \in U.$$

Definition. It is said to be a bitripotent of U if a pair (c_1, c_2) of the tripotents satisfy the relations

$$< c_1c_1c_2 >= \alpha c_2, < c_2c_2c_1 >= \alpha c_1,$$

$$< c_1c_2c_1 >= \beta c_2, < c_2c_1c_2 >= \beta c_1,$$

$$< c_2c_1c_1 >= \gamma c_2, < c_1c_2c_2 >= \gamma c_1,$$

and other products are zero, where

$$\alpha^2 + \beta^2 + \gamma^2 \neq 0, \alpha, \beta, \gamma \in \mathbf{R}.(\text{real number})$$

3 Several examples

In this section, we give several examples induced from certain bilinear forms.

Example 2.1 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= \overline{< y, x >}$, $< x, \bar{y} >=< y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >=< x, y > z + < z, y > x - < z, \bar{x} > \bar{y}.$$

is a hermitian (-1,1)Jordan triple system.

Example 2.2 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= -\overline{< y, x >}$, $< x, \bar{y} >= - < y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >=< x, y > z - < z, y > x - < z, \bar{x} > \bar{y}.$$

is a hermitian (1,-1)Jordan triple system.

Example 2.3 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= \overline{< y, x >}$, $< x, \bar{y} >=< y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >=< x, y > z - < z, y > x.$$

is a hermitian (-1,-1)Jordan triple system.

Example 2.4 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $\overline{< x, y >} =< \tilde{x}, \tilde{y} >=< y, x >$, $< x, \tilde{y} >= - < y, \tilde{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$ with $\tilde{x} = -x$. Then*

$$< xyz >= - < x, y > z + < z, y > x - < z, \tilde{x} > \tilde{y}.$$

is a hermitian (-1,-1)Jordan triple system. If we define a bilinear form as $< x, y >= -x_1 \cdot \bar{y}_1 - x_2 \cdot \bar{y}_2$ for $x = (x_1, x_2)$ and $\tilde{x} = (\bar{x}_2, -\bar{x}_1)$,where x_1 and x_2 are N-vectors, this system is called $sp(2N) \oplus u(1)$ hermitian 3-algebra in M-theory [39, 41, 42].

Example 2.5 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= -\overline{< y, x >}$, $< x, \bar{y} >= - < y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >=< x, y > z + < z, y > x.$$

is a hermitian (1,1)Jordan triple system.

Next we consider the case of $K(x, y) \neq 0$.

Proposition 2.6 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= -\varepsilon\overline{< y, x >}$, $< x, \bar{y} >= -\varepsilon < y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >=< z, y > x.$$

is a hermitian (ε, δ) Freudenthal-Kantor triple system.

Proposition 2.7 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= -\overline{< y, x >}$, $< x, \bar{y} >= - < y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >= 1/2(- < x, y > z - < z, y > x + < x, \bar{z} > \bar{y}).$$

is a hermitian (1,1)Freudenthal-Kantor triple system with $K(x, z)y =< x, \bar{z} > \bar{y}$.

Proposition 2.7 *Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= \overline{< y, x >}$, $< x, \bar{y} >=< y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then*

$$< xyz >= 1/2(< x, y > z - < z, y > x + < x, \bar{z} > \bar{y}).$$

is a hermitian (-1,-1)Freudenthal-Kantor triple system with $K(x,z)y =< x,\bar{z} > \bar{y}$.

Proposition 2.8 ([11]) *Let U be a (-1,1)Freudenthal-Kantor triple system with a left unit unit element u (i.e, L(u,u)=Id). Then we have a decomposition,*

$$U = U_1 \oplus U_3$$

where $U_i = \{x| \quad < xuu >= ix \}$.

Furthermore, we have

$$U_1 = U_1^+ \oplus U_1^-,\ U_3 = U_3^+ \oplus U_3^-,$$

where

$$Q(x) :=< uxu >= \pm x \quad if\ x\ \in U_1^{\pm}, Q(x) :=< uxu >= \pm 3x \quad if\ x\ \in U_3^{\pm}.$$

Conversely, if we set $x \cdot y =< xuy >$, *then the triple product* $< xyz >$ *can be expressed in terms of* $x \cdot y$;

$$< xyz >= (Q^{-1}(y) \cdot x) \cdot z + x \cdot (Q^{-1}(y) \cdot z) - Q^{-1}(y) \cdot (x \cdot z),$$

Remark We note that for accurate example of above bilinear forms,

$$< x,y >= x_1\bar{y}_1 + \cdots + x_n\bar{y}_n \Rightarrow\ \overline{< x,y >} =< y,x > .$$

where $x = (x_1, \cdots, x_n),\ y = (y_1, \cdots, y_n)$,

or

$$< x,y >= x_1\bar{y}_2 - x_2\bar{y}_1 + \cdots + x_{2n-1}\bar{y}_{2n} - x_{2n}\bar{y}_{2n-1} \Rightarrow\ \overline{< x,y >} = - < y,x >,$$

where $x = (x_1, \cdots, x_{2n}),\ y = (y_1, \cdots, y_{2n})$.

Let U be a set of $Mat(1,n;\mathbf{C})$ for their triple products induced in this section and $x = \Sigma_i(\lambda_i + \mu_i\sqrt{-1}) \quad \in U$, where $\lambda_i,\ \mu_i$ are real numbers, if we set

$$||x|| = max(|\lambda_i|, |\mu_i|),$$

then we have a hermitian property, because $|| < xxx > || \ \leq K||x||^3$ and for $\varepsilon = \pm 1$ it holds $L(x,y)^* = -\varepsilon L(y,x)$ and $(L(x,y)z,w) = (z, L(x,y)^*w)$.

For a special example of Proposition 2.6 ($\varepsilon = -1, \delta = 1$and $dim_{\mathbf{C}}\ U = n$), we have a tripotent element as follows:

If e_i and e_j are basis of U satisfying $< e_i, e_j >= \delta_{ij}$, then e_i and e_j are bitripotents of U, because $< e_je_ie_i >= e_j, < e_ie_je_i >=< e_ie_ie_j >= \delta_{ij}e_i$.

4 Other triple systems

In this section, we give another examples without generalized Jordan triple systems and examples of nonassociative algebras given by our triple systems, furthermore exhibit an image of geometrical view point.

Example 3.1

Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= \overline{< y, x >}$, $< x, \bar{y} >=< y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then

$$< xyz >=< z, y > x + < z, \bar{x} > \bar{y}.$$

is a hermitian triple system satisfying the relation

$$< ab < xyz >>=<< abx > yz > + < x < bay > z > - < xy < abz >> .$$

Example 3.2

Let U be a vector space equipped with a bilinear form $< x, y >$ such that $< x, y >= -\overline{< y, x >}$, $< x, \bar{y} >= - < y, \bar{x} >$ and $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$. Then

$$< xyz >=< z, y > x - < z, \bar{x} > \bar{y}$$

is a hermitian triple system satisfying the relation

$$< ab < xyz >>=<< abx > yz > - < x < bay > z > - < xy < abz >> .$$

Remark 3.3 *Let U be a vector space over $\mathbf{C}$ with a hermitian bilinear form $< x, y >= \overline{< y, x >}$, $< \alpha x, \beta y >= \alpha\bar{\beta} < x, y >$ and an element e such that $< e, e >= 1$ and $\bar{e} = e \in U$*

If we define the binary product by

$$x \circ y =< y, e > x$$

then binary algebra $(U, x \circ y)$ is a noncommutative Jordan algebra satisfying the flexible law, i.e.,

$$(x \circ y) \circ x = x \circ (y \circ x)$$

$$x \circ (y \circ x^2) = (x \circ y) \circ x^2$$

and have a right unit element , i.e.,

$$e \circ e = e, \quad x \circ e = x \quad but \quad e \circ x \neq x.$$

Indeed, we calculate

$$(x \circ y) \circ x = (< y, e > x) \circ x =< y, e >< x, e > x.$$

On the other hand, we have

$$x \circ (y \circ x) = x \circ (< x, e > y) =< x, e >< y, e > x.$$

Hence we obtain

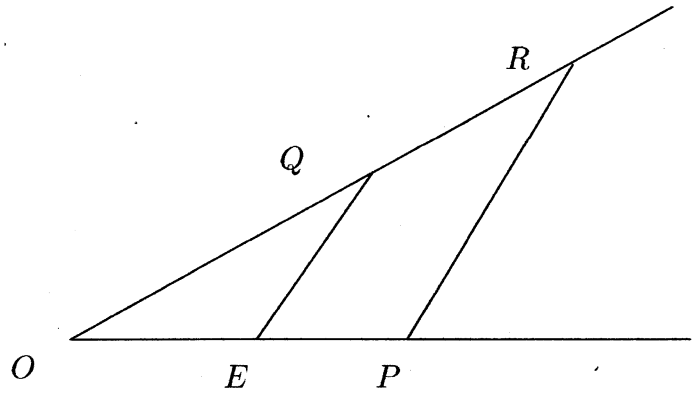
$$x \circ (y \circ x) = (x \circ y) \circ x.$$

Furthermore,by straight forward calculations, we have

$$(x \circ x) \circ y = x \circ (x \circ y), \quad (y \circ x) \circ x = y \circ (x \circ x), \quad (alternative \ \ law)$$

$$(x, y, x^2) = 0 \quad (Jordan \ \ property)$$

In geometrical view point roughly, we may make an image as follows:



$$\frac{|\overrightarrow{OR}|}{|\overrightarrow{OQ}|} = \frac{|\overrightarrow{OP}|}{|\overrightarrow{OE}|} \Longrightarrow \overrightarrow{OR} =< y, e > x,$$

where $\overrightarrow{OQ} = x$, $\overrightarrow{OE} = e$, $|\overrightarrow{OE}| = 1$, $\overrightarrow{OP} = y$.

Indeed, from the fact that $|\overrightarrow{OP}||\overrightarrow{OE}| =< y, e >$, $< e, e >= 1$, it follows the result.

5 Application to field theory

In this section, we apply a hermitian triple system to a field theory.

We start with

$$S = \int d^3x \mathrm{tr}(-L(\mathbf{D}_\mu Z, \mathbf{D}^\mu Z) + V(Z) \\ + \epsilon^{\mu\nu\lambda}(-\frac{1}{2} A_{\mu bc}\partial_\nu A_{\lambda da} L(< T^c T^b T^a >, T^d) \\ + \frac{1}{3} A_{\mu da} A_{\nu bc} A_{\lambda fe} L(< T^c T^b T^a >, < T^f T^e T^d >))), \quad (1)$$

where

$$\mathbf{D}_\mu Z = \partial_\mu Z - A_{\mu ba} < T^a T^b Z > . \quad (2)$$

Z and A_μ are matter and gauge fields, respectively. $V(Z)$ is a potential term. This action is invariant under the transformations generated by the operator $L(x,y) - L(y,x)$. The action (1) equipped with a Lorentzian Lie 3-algebra and a hermitian 3-algebra describes the bosonic parts of the effective actions of supermembranes in M-theory. We apply a hermitian triple system, $< xyz >= (x \cdot \bar{y})z + (z \cdot \bar{y})x - (z \cdot x)\bar{y}$ in Example 2.1 with a bilinear form $< x, y >= (x \cdot \bar{y})$ for vectors x and y, to this action and see what kind of field theory we obtain. The metric is given by $\mathrm{tr}L(x,y) = (x \cdot \bar{y})$ after an appropreate renormalization.

The covariant derivative is explicitly written down as

$$\mathbf{D}_\mu Z = \partial_\mu Z - iA^L_\mu Z - iZA^R_\mu, \quad (3)$$

where $A^L_\mu := -iA_{\mu ba}(T^a \cdot \bar{T}^b)$ generate the $u(1)$ Lie algebra, whereas $A^R_\mu := -iA_{\mu ba}(\cdot \bar{T}^b T^a - \cdot T^a \bar{T}^b)$. In order to see how gauge transformations act on these fields, let us see how $L(x,y) - L(y,x)$ acts on the coupling between A_μ and Z:

$$\begin{aligned} & \delta(A_{\mu ba} < T^a T^b Z >) \\ = \; & \Lambda_{dc} A_{\mu ba}(<< T^c T^d T^a > T^b Z > - < T^a < T^d T^c T^b > Z > \\ & \qquad + < T^a T^b < T^c T^d Z >>) \\ = \; & -A^L_\mu(\Lambda^L Z) - A^L_\mu(Z\Lambda^R) + Z[\Lambda^R, A^R_\mu] - (\Lambda^L Z + Z\Lambda^R)A^R_\mu, \end{aligned} \quad (4)$$

where gauge parameters Λ^R and Λ^L are defined in the same way as A^R_μ and A^L_μ, respectively. From this, we obtain gauge transformations,

$$\begin{aligned} \delta A^R_\mu &= \partial_\mu \Lambda^R + i[A^R_\mu, \Lambda^R] \\ \delta A^L_\mu &= \partial_\mu \Lambda^L \qquad (5) \\ \delta Z &= i\Lambda^L Z + Z(i\Lambda^R). \qquad (6) \end{aligned}$$

Therefore, A^R_μ and A^L_μ transform as adjoint representations, whereas Z transforms as a bi-fundamental representation. As a result, the action can be rewritten in a covariant form and we obtain a Chern-Simons gauge theory,

$$\begin{aligned} S &= \int d^3x \mathrm{tr}(-(\partial_\mu Z - iA^L_\mu Z - iZA^R_\mu) \cdot \overline{(\partial_\mu Z - iA^L_\mu Z - iZA^R_\mu)} \\ &\quad + \epsilon^{\mu\nu\lambda}(\frac{1}{4}(2A^L_\mu \partial_\nu A^L_\lambda + A^R_\mu \partial_\nu A^R_\lambda) - \frac{i}{6} A^R_\mu A^R_\nu A^R_\lambda)). \qquad (7) \end{aligned}$$

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SOME CLARIFICATION AND CORRECTION OF RECENT CRITICISM OF EINSTEIN

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Abstract

Although the mathematics involved is fairly straightforward, the full derivations of two crucial equations in Einstein's 1905 article, *On the Electrodynamics of Moving Bodies* are presented to aid clarification but, more particularly, to correct a misapprehension, which has been afforded some recent publicity, that some trickery is employed by Einstein in his seminal paper.

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In my book 'Exploding a Myth'[1], I clearly express some degree of scepticism concerning the almost pre-eminent place seemingly afforded the theories of both special and general relativity in present day physics. This is not because I believe them to be necessarily incorrect but because, almost always, the results may be derived by alternative methods. Also, I believe I was one of the first people to modify the traditional derivation of Maxwell's electromagnetic equations to cover the more general case of a moving medium [2]. These modified equations are easily seen to be invariant under Galilean transformation and so, one must wonder at the consequences for physics if Maxwell himself had done this. One must wonder, in fact, if what we know today as special relativity would ever have come about? In general relativity, virtually all the results associated with the so-called crucial tests of that theory have been derived without using relativistic ideas by, for example, my friend and colleague Bernard Lavenda [3]. In any case, it might be remembered that the accepted result for the shift of the perihelion of Mercury had been derived by Gerber in 1898 [4] long before the advent of general relativity but not recognised for some unexplained reason.

In my view, therefore, it is perfectly valid for people to question many of the results of both special and general relativity. However, many of the results associated with relativity have been shown to be valid as far as our present level of understanding is concerned and so one must be very careful when criticizing accepted results. Relativity theory is essentially mathematical in nature and, therefore, when criticizing this mathematics, it is vital that no mathematical mistakes appear in that criticism. This is manifestly not so in the case of at least some of the manipulations in the article 'Experimental and Theoretical Evidences of Fallacy of Space-time Concept and Actual State of Existence of the Physical Universe'[5].

The prime example here must be the so-called 'equation of trickery', although in Khan's article it is not absolutely clear to which equation, or equations, this term refers. The two equations in question appeared in one of Einstein's 1905 papers – specifically *On the Electrodynamics of Moving Bodies* [6]. Here, firstly, the more basic of these equations is derived correctly precisely as Einstein himself clearly indicated: after a time t has elapsed, the x-coordinates in the two frames are related by

$$x' = x - vt$$

where the dashed coordinates refer to the moving frame of reference.

Hence,

$$x = x' + vt$$

and so, if a light ray takes time t to travel from the origin of the stationary frame to the point x then the speed taken is given by

$$x/t \text{ or } (x' + vt)/t$$

and, if the speed of light is constant and given by c, both of these expressions must equal c. It follows immediately from putting the second expression equal to c that

$$t = x'/(c - v)$$

If the ray is moving in the opposite direction, the time taken is given by

$$t = x'/(c + v)$$

Hence, assuming the speed of light to be constant and inserting the arguments of the function τ into the equation

$$(\tau_0 + \tau_2)/2 = \tau_1,$$

results precisely in Einstein's equation:

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,t+\frac{x}{c-v}+\frac{x}{c+v}\right)\right]=\tau\left(x,0,0,t+\frac{x}{c-v}\right)$$

and so, no trickery is involved in deriving this equation if you accept the validity of the principle of the constancy of the speed of light.

However, at this point Khan [7] appears to take further issue with Einstein and refer to his following equation as the 'Equation of Trickery'. But is it? On subtracting $\tau(0,0,0,t)$ from both sides, the above equation may be written

$$\frac{1}{2}\left[\tau\left(0,0,0,t+\frac{x}{c-v}+\frac{x}{c+v}\right)-\tau(0,0,0,t)\right]=\tau\left(x,0,0,\frac{x}{c-v}\right)-\tau(0,0,0,t)$$

(1)

and the right-hand side of this equation may be replaced by

$$\tau\left(x,0,0,\frac{x}{c-v}\right)-\tau\left(0,0,0,\frac{x}{c-v}\right)+\tau\left(0,0,0,\frac{x}{c-v}\right)-\tau(0,0,0,t) \quad .$$

Following Einstein, if x is taken to be infinitesimally small, then $\frac{x}{c-v}$ and $\frac{x}{c-v}+\frac{x}{c+v}$ will be *infinitesimally small* also. Consequently, in the limit as $x \to 0$, the left-hand side of (1) *is* seen to take the form

$$-\frac{1}{2}\left[\frac{x}{c-v}+\frac{x}{c+v}\right]\frac{\partial\tau}{\partial t} \quad ,$$

the first two terms of the expanded right-hand side give

$$-x\frac{\partial\tau}{\partial x} \quad ,$$

and the final two terms give

$$\frac{-x}{c-v}\frac{\partial\tau}{\partial t} \quad ,$$

if the usual elementary definitions commonly used in differential calculus are employed.
Hence, combining these latter three expressions gives the equation

$$\frac{1}{2}\left[\frac{1}{c-v}+\frac{1}{c+v}\right]\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x}+\frac{1}{c-v}\frac{\partial\tau}{\partial t}\quad .$$

It follows that there is absolutely no trickery involved in deducing this equation, just simple straightforward differential calculus and this equation is correct, although Einstein doesn't include the actual manipulation in his article.

Hence, the stated basis for the article, 'Experimental and Theoretical Evidences of Fallacy of Space-time Concept and Actual State of Existence of the Physical Universe', is seen to be false and this has to raise serious questions concerning the remainder.

Various other points concerned with Khan's article [5] should be noted also. Energy/mass equivalence has been found valid experimentally on occasions too numerous to mention, so to criticise it is a highly questionable line to follow. Also, in the present context, it is important to note that, although this result is usually attributed to Einstein, it dates from the previous century where the notion was originally both derived and used. Again, it might be remembered usefully that the whole idea of mass/energy equivalence originated in electromagnetic theory as may be seen from, for example, J.J.Thomson's book *Electricity and Matter* [8], although the identification itself dates from the earlier century. This, of course, goes back to a time when investigators were attempting to explain the force of gravity in terms of the electromagnetic force. Khan also makes several comments about the famous Michelson- Morley experiment. Here, however, attention must be drawn to the fact that the well-known problems associated with that experiment have been investigated in minute detail, particularly in more recent years by R. Cahill, whose work is readily available for scrutiny via the internet. Without reference to this fairly recent work, no sensible comment may be made, in my view, about questions concerning that historically important experiment.

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SEVERAL ROUTES TO DETERMINING ENTROPY GENERATION IN THE EARLY UNIVERSE/LINKS TO CMBR SPECTRA/RELIC NEUTRINO PRODUCTION

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Abstract

We analyze how entropy could be generated via a semi-classical argument as well as by multiple brane - anti brane combinations leading to an initial soliton- instanton formation. The supposition is that the two different types of methods give similar initial conditions for entropy andinformation/ computational bits of information in the initial universe. We close then with observations we think are pertinent to entropy increase and also the variation of statistical noise about the CMBR spectra. This ties in with possible new species of detectable 'neutrinos' in ways which lead to an extension of the standard model, since the derived 'axion' is coupled to photons to the tune of $f_a = \vartheta\left(10^{15} GeV\right)$ which is too large for Earthbound experiments but which is in the range of earth bound experiments, as noted by P. Pugnat et al, but which could conceivably be detected by proper analysis of the CMBR spectra

Keywords: Soliton-instanton formation, Neutrinos, CMBR ,

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1 Introduction

We wish to present two alternative routes to generation of entropy. One is a semi classical argument, linked to the exponential of early universe entropy being proportional to the spatial integral of energy minus an interaction between particles or clumps. The other is a brane world summation of brane and anti brane components which are in very small space time geometry of no less than several orders of magnitude larger than Planck length, cubed. This second result is orthodox brane theory, but this same argument as given pre supposes the same increase in minimum geometrical length scale which the more semi classical argument gives.

2. Semi classical models of entropy generation.

Kolb and Turner [2]have a temperature T related entropy density which leads to that we are able to state total entropy as the entropy density time's space time volume
V_4.

$$S_{total} \equiv s_{Density} \cdot V_4 = \frac{2\pi^2}{45} \cdot g_{\bullet} \cdot T^3 \cdot V_4 \tag{1}$$

$T \approx 10^{32} K$. We can compare this with Thanu Padamanadan's [7] treatment of entropy which is with regards to micro canonical ensemble as defined via

$$\exp(S_{total}) = g(E) = \frac{A}{N!} \cdot \int d^{3N}x \cdot \left[E - \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j) \right]^{\frac{3N}{2}} \tag{2}$$
$$\leq \frac{A}{N!} \cdot \int d^{3N}x \cdot [E]^{\frac{3N}{2}} \approx \left[\frac{A}{N!} \cdot \int d^{3N}x \right] \cdot \left[\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \right]^{\frac{3N}{2}}$$

This gives us the option of comparing what we get in entropy with Seth Lloyds[3]

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = \left[\rho \cdot c^5 \cdot t^4 / \hbar\right]^{3/4} \qquad (3)$$

We can if we take the absolute value of (3) and (2) above get for small volume values good estimates as to the relative volume of the phase space in early universe cosmology where (2) and. (3) give similar entropy values and we assume $\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 >> \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$ [1], where U is a potential energy of self interacting particles in an early universe cosmology.

3. Brane world picture of early universe entropy formation

This is adapted from a lecture at ICGC-07 by Mathur [4]. We propose that branes and anti branes form the working component of an instanton. I.e. we obtain for D space time dimensions, and E the general energy $S \sim E^{(D-1/D)}$. Note that Mathur's classical and quantum gravity article[5] has a string winding interpretation of energy along the lines of putting energy E into string windings which leads to an entropy defined in terms of an energy value of, if mass $m_i = T_P \prod\prod L_j$ (with T_P being the tension of the i th brane, and L_j being spatial dimensions of a complex torus structure) This leads to entropy [4,5]

$$S_{Total} = A \cdot \prod_{i}^{N} \sqrt{n_i} \tag{4}$$

Our claim is that this very specific value of entropy for (4) above will at about the onset of inflation lead to [1,3]

$$\left| \left[S_{Total} = A \cdot \prod_{i}^{N} \sqrt{n_i} \right] \Big/ k_B \ln 2 \right| \approx [\#operations]^{3/4} \approx 10^8 \tag{5}$$

Furthermore we claim that the interaction of the branes and anti branes will form an instanton structure, which is implicit in the treatment outlined in (5), and that the numerical counting given in (4) merely reflects that branes and anti branes, even if charge conjugates of each other have the same 'wrapping number' n_i.How to tie in the entropy with the growth of the scale function? Racetrack models of inflation, provide a power spectrum given by [1]

$$P \sim \frac{1}{150\pi^2} \cdot \frac{V(\phi)}{\epsilon} \tag{6}$$

This is assuming a slow roll parameter treatment with $\epsilon << 1$, and $t > t_P$. An increase in scalar power, is then proportional to an increase in entropy via [1]

$$\left| \frac{\Delta E}{l_P^3} \right| \sim \left| \frac{\Delta P \, \epsilon \, 150\pi^2}{l_P^3} \right| \approx |\Delta S| \tag{7}$$

4 . Neutrino physics issues and changes from the SUSY paradigm

K. Meissner and H. Nicolai [6] recently postulated an extension of the SM (standard model) involving a classically conformal Langrangian. The outstanding feature of their model is that if we extend the standard model the way they intend to with the usual Higgs doublet Φ and one extra weak scalar field $\widetilde{\phi}$ we write as[6]

$$\widetilde{\phi}(x) = \varphi(x) \cdot \exp\left[\frac{ia(x)}{\sqrt{2}\mu}\right] \qquad (8)$$

Where the field $a(x)$ then gives rise to a (pseudo-) Goldstone particle associated "with the spontaneous breaking of a new global $U(1)_L$ (modified Lepton number) symmetry [6]. This boson, the so called 'Majoran' shares many properties with the axion" . And furthermore we use conformal symmetry to eliminate in their conformal Lagrangian contributions from all but $\widetilde{\phi}^4$ terms in their Lagrangian so eventually we have masses for particles like 'neutrinos' which are heavier than the SUSY neutrino candidate, but with the same 'branching ratio " for a particle signature which is like the Higgs but with a lower cross section".

Quoting what was said in the abstract. This ties in with possible new species of detectable neutrinos in ways which lead to an extension of the standard model, since the derived 'axion'[6]is coupled to photons to the tune of $f_a = \vartheta(10^{15} GeV)$ which is too large for Earthbound experiments but which is in the range of astro physical measurements/ experiments, as noted by P. Pugnat et al [8], which could conceivably be detected by proper analysis of the CMBR spectra. If we look at the power spectra as noted in (6) and its links to

fluctuations of entropy, we may be able to obtain the necessary conditions for obtaining a signature for how the 'Majoran' [6], i.e. the retro fitted axion, is coupled to photons, while leaving to the LHC determination of actual masses of 'particles' so generated

5. Conclusion

Now, how does this tie in with the lumpiness seen in the CMBR spectra? In an e mail communication, Subir Sarkar [9] summarized the situation up as follows. "Quasi-DeSitter spacetime during inflation has no "lumpiness" - it is necessarily very smooth. Nevertheless one can generate structure in the spectrum of quantum fluctuations originating from inflation by disturbing the slow-roll of the inflaton in our model this happens because other fields to which the inflaton couples through gravity undergo symmetry breaking phase transitions as the universe cools during inflation." We intend to examine how this is linkable to entropy variations in (7) in future numerical simulations of CMBR irregularities. Furthermore, we also should attempt to find traces for the 'axion' photon coupling in careful review of the CMBR and use it as part of extending the standard model to accommodate relic early universe neutrino physics without having to invoke SUSY, Technicolor, and some of the other constructions of theoretical physics.

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HADRONIC JOURNAL 37, 163 - 177 (2014)

FOUNDATIONS OF ISOMATHEMATICS

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Abstract

This paper is a step by step logical account of the origin of Isomathematics as developed by Rugero Maria Santilli [10],[11], [12].

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1 Introduction

Modern mathematics has a strong foundation laid down by Dr.Bertrand Russell and Dr.Whitehead through 'Principia Mathematica'. Algebraic structures like Group, Ring, Field, Algebra, Vector space and related structures have already found vast applications in the modern mathematical and scientific developments. More general structures like Groupoids, Semigroups, Monoids, Quasigroups and Loops were also being studied in the 20th century, which were to find vast applications in future. The detailed consolidated account of these generalized structures is found in 'Survey of Binary Systems' by R.H.Bruck [1] in 1958.

While the scientific discoveries and mathematical knowledge were moving hand in hand, towards the end of 20th century there were few mathematically unexplained physical phenomena in Quantum mechanics and Quantum Chemistry. These situations called for a more generalized mathematical structure, latter called as 'Isofield'.

Broadly speaking there are two kinds of Dynamical systems. First one is the conventional 'Exterior Dynamical system' and the second one is the more complex but generalized 'Interior Dynamical system'.

2 Dynamical Systems

System
Exterior Dynamical Systems. Point-like particles are moving in a homogeneous and isotropic vacuum with local-differential and potential-canonical equations of motion.

$\longmapsto$ **Nature**
Linear, Local, Newtonian Lagrangian and Hamiltonian.

$\longmapsto$ **Mathematics**
Conventional Mathematical Structures such as Algebras, Geometries, Analytical Mechanics, Lie Theory.

System
Interior Dynamical Systems. Extended non-spherical deformable particles moving within non-homogeneous anisotropic physical medium.

$\longmapsto$ **Nature**
Non-linear, non-local, non-Newtonian, non-Lagrangian and non-Hamiltonian.

$\longmapsto$ **Mathematics**
Non-conventional most general possible mathematical structures which are axiom preserving non-linear and non-local formulations of current mathematical structures.

2.1 Santilli's Achievement

Problem		**Solution**		**Span**
To represent the non-local, non-linear, non-Lagrangian, non-Hamiltonian, and non-Newtonian system characterizing the motion of extended particle within physical media.	$\longmapsto$	Isotopic Generalization of contemporary mathematical structures, like Field, Vectorspace, Transformations, Lie Algebra and conventional geometries.	$\longmapsto$	The Isotopic generalization by Santilli leads to the maximum generalization of Gallili's and Einstein's relativities and contemporary mathematical structures.

2.2 Emergence of New Mathematics

It was Santilli in early 1992 who first discovered the concept of 'Isofield' which further led to a plethora of new isoalgebraic structures and a whole new 'Isomathematics' which is a step further in Modern Mathematics.

This work aims at exploring the very basics of Isomathematics as formulated by Santilli [8] and [12]. The concept of 'Isotopy' plays a vital role in the development of this new age mathematics. Starting with Isotopy of groupoids we develop the study of Isotopy of quasi groups and loops via Partial Planes, Projective planes, 3-nets and multiplicative 3-nets.

3 From Partial Plane to Loop

Definition 3.1. *A **partial plane** is a system consisting of a non-empty set G partitioned into two disjoint subsets(one of which may be empty), namely the point-set and the line-set together with a binary relation, called incidence, such that (i) (Disjuncture) If x is incident with y in G then one of x, y is a line of G and the other is a point, (ii) (Symmetry) If x is*

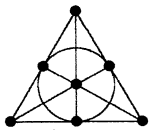
incident with y in G then y is incident with x in G, and (iii) If x, y are distinct elements of G there is at most one z in G such that x and y are both incident with z in G.

Definition 3.2. *A **Projective plane** is a special kind of a partial plane G such that; (iv) if x and y are distinct points or distinct lines of G, there exists a z in G such that x and y are both incident with z in G ; (v) there exists at least one set of four distinct points of G no three of which are incident in G with the same element.*

It is easy to show that in the presence of (i)- (iv), postulate (v) is equivalent to ; (vi) there exists at least one set of four distinct lines of G no three of which are incident in G with the same element.

A *projective plane* of order n has
$n^2 + n + 1$ points;
$n^2 + n + 1$ lines;
$n + 1$ points on each line;
$n + 1$ lines through each point.

Example: The projective plane of order $n = 2$



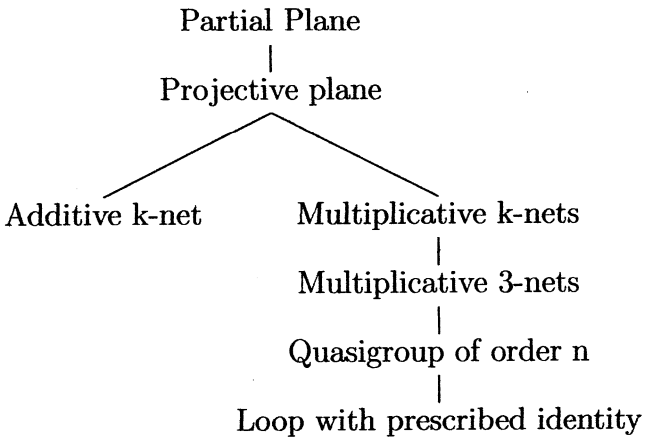
7 points
7 lines
3 points on each line
3 lines through each point

3.1 k-net

Definition 3.3. *A **k-net** is a partial plane N whose line-set has been partitioned into k disjoint classes such that (a) N has at least one point, (b) Each point of N is incident in N with exactly one line of each class, and (c) Every two lines of distinct classes in N are both incident in N with exactly one point.*

If some line of a k-net N is incident with exactly n distinct points in N, so is every line of N. The cardinal number n is called the **order of N.**

Since a net is a partial plane, every net may be embedded in at least one projective plane. Every projective plane contains nets and, of these, two types additive 3-net and multiplicative 3-net have special significance. The following tree diagram is self-explanatory.



4 From Geometry to Algebra

4.1 From 3-net to Loop

We can have an **additive 3-net** and **multiplicative 3-net** of a projective plane.

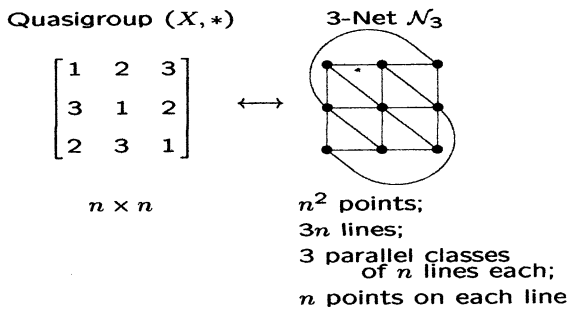
Definition 4.1. *A **Quasigroup** is a groupoid G such that, for each ordered pair $a, b \in G$, there is one and only one x such that $ax = b$ in G and one and only one y such that $ya = b$ in G.*

In other words a quasigroup is groupoid whose composition table is a Latin square.

Definition 4.2. *A **loop** is a quasigroup with an identity.*

An associative loop is a group.

A quasigroup $(X, *)$ of order n determines a *3-net*, eg.



- *Every 3-net N of order n gives rise to a class of quasigroups $(Q, \circ)$ of order n* by defining one-to-one mappings $\theta(i)$ with $i = 1, 2, 3$ of Q upon the class of i-lines of N.
- Two quasigroups obtainable from the same 3-net by different choices of the set Q or of the mappings $\theta(i)$ are said to be **isotopic**.
- *For any Q, the $\theta(i)$ can be so chosen that $(Q, \circ)$ is a loop with a prescribed element e of Q as identity element.*

5 Isotopy of Groupoids

Definition 5.1. *Let $(G, \cdot)$ and $(H, \circ)$ be two groupoids. An ordered triple (α, β, γ) of one-to-one mappings α, β, γ of G upon H is called an* **isotopism**

of $(G, \cdot)$ *and* $(H, \circ)$. *provided* $(x\alpha) \circ (y\beta) = (x.y)\gamma$.
$(G, \cdot)$ *is said to be isotopic with* $(H, \circ)$ *or* $(G, \cdot)$ *is said to be an* **isotope** *of* $(H, \circ)$.

Isotopy of groupoids is an equivalence relation. Every isotope of a quasigroup is a quasigroup.

5.1 Origin of Isotopy

We underline the importance of 'Isotopy' by borrowing two paragraphs from the book 'A Survey of Binary Systems' by R.H.Bruck [1].

"The concept of isotopy seems very old. In the study of Latin squares (which were known to BACHET and certainly predate Euler's problem of the 36 0fficers) the concept is so natural to creep in unnoticed; and Latin squares are simply the multiplication tables of finite quasigroups." "It was consciously applied by SCHÖNHART, BAER and independently by ALBERT. ALBERT earlier had borrowed the concept from topology for application to linear algebras; in the latter theory it has virtually been forgotten except for applications to the theory of projective planes."

6 Examples on Isotopy of Groupoids

Consider the two groupoids $G = \{1, 2, 3\}$ and $G' = \{a, b, c\}$ defined by the following composition tables.

.	1	2	3
1	1	3	2
2	3	1	3
3	2	3	2

and

*	a	b	c
a	a	c	b
b	b	b	c
c	a	a	b

Then the ordered triple (α, β, γ) defined by the permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 \\ b & c & a \end{pmatrix}$ and $\gamma = \begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix}$ is an isotopy.

Thus $(G,.)$ and $(G,*)$ are **isotopic.**
Note that an isomorphism is just a particular case of isotopy wherein $\alpha = \beta = \gamma$. If l is the identity mapping then (α, β, l) is called a ***principal isotopy*** between the two groupoids.

6.1 Isotopy of Quasigroups

Consider the groupoids $(L,.)$ and $(L',*)$ with multiplication tables as;

.	0	1	2	3	4
0	0	1	3	4	2
1	1	0	2	3	4
2	3	4	1	2	0
3	4	2	0	1	3
4	2	3	4	0	1

and

*	0	1	2	3	4
0	1	0	4	2	3
1	3	1	2	0	4
2	4	2	1	3	0
3	0	4	3	1	2
4	2	3	0	4	1

Here the ordered triple (α, β, γ) defined as $\alpha = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 0 & 3 \end{pmatrix}$ and $\gamma = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 0 & 4 & 3 \end{pmatrix}$ is an Isotopism. Note that L and L' are quasi groups.

6.2 Principal Isotopy of Groupoids

Consider the two groupoids G and G' defined by the following composition tables.

.	1	2	3
1	1	3	2
2	3	1	3
3	2	3	2

and

*	1	2	3
1	1	2	2
2	3	2	1
3	1	3	3

Then the ordered triple (α, β, γ) defined by the permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and $\gamma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ is a principal isotopy.

Consider the groupoids and their isotopy as defined in Example 1, we can define $\delta = \alpha\gamma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $\eta = \beta\gamma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and $l = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$.

Note that (δ, η, l) is the principal isotopy corresponding to (α, β, γ). ***In this manner every isotopy gives rise to a principal isotopy such that the isotope and the principal isotope are isomorphic.***

In general, it is sufficient to consider only the principal isotopies in view of the following Theorem.

Theorem 6.1. *If G and H are isotopic groupoids then H is isomorphic to a principal isotope of G.*

Proof. *Let (α, β, γ) be an isotopy of G on to H. Let $\delta = \alpha\gamma^{-1}$ and $\eta = \beta\gamma^{-1}$. We have $(\alpha, \beta, \gamma) = (\delta\gamma, \eta\gamma, \gamma)$. Hence there exists a groupoid K such that (δ, η, i) is a principal isotopy of G on to K and γ is an isomorphism of K on to H.*

'Necessary and sufficient conditions that a groupoid possess an isotope with identity element are that the groupoid have a right nonsingular element and a left nonsingular element' Ref.[1]p.57.

All the elements of a quasigroup are left nonsingular and right nonsingular (as every element occurs only once in every row and column). Therefore **every quasigroup is isotopic to a loop.**

- **This lifting of the multiplicative quasigroup to a loop with the prescribed identity gives rise to a multiplicative group with the 'prescribed identity' which Santilli termed as 'Isounit'. The resulting field with the multiplicative isounit is called as an Isofield [4].**
- Without loss of generality we can say that the words 'Isotopy' and 'Axioms preserving' are synonymous.

7 Santilli's Isofield

Definition 7.1. *Given a field F with elements $\alpha, \beta, \gamma, \ldots$, sum $\alpha + \beta$, multiplication $\alpha\beta$, and respective units 0 and 1, "Santilli's isofields" are rings of elements $\hat{\alpha} = \alpha\hat{1}$ where α are elements of F and $\hat{1} = T^{-1}$ is a positive-definite $n \times n$ matrix generally outside F equipped with the same sum $\hat{\alpha} + \hat{\beta}$ of F with related additive unit $\hat{0} = 0$ and a new multiplication $\hat{\alpha} * \hat{\beta} = \hat{\alpha} T \hat{\beta}$, under which $\hat{1} = T^{-1}$ is the new left and right unit of F in which case $\hat{F}$ satisfies all axioms of a field.*

The 'isofields' $\hat{F} = \hat{F}(\hat{\alpha}, +, \hat{\times})$ are given by elements $\hat{\alpha}, \hat{\beta}, \hat{\gamma} \ldots$ characterized by one-to-one and invertible maps $\alpha \to \hat{\alpha}$ of the original element $\alpha \in F$ equipped with two operations $(+, \hat{\times})$, the conventional addition $+$ of F and a new multiplication $\hat{\times}$ called "isomultiplication" with corresponding conventional additive unit 0 and a generalized multiplicative unit $\hat{1}$, called "multiplicative isounit" under which all the axioms of the original field F are preserved.

If the conventional field is chosen to be alternative under the operation of conventional multiplication then the resulting isofield is also isoalternative under isomultiplication.
If the given algebraic structure is a noncommutative division ring (e.g. ring of quaternions) then the resulting isoalgebraic structure is also noncommutative under the isomultiplication.

This new algebraic structure has revolutionized contemporary mathematics and found its applications in so far unexplored (unexplained) and unknown territories of quantum mechanics and quantum chemistry.

The resulting new theory of numbers has become the basis of recent studies of **nonlinear-nonlocal, non-Hamiltonian systems** in nuclear particle and statistical physics.

7.1 Isofields and Isonumbers

Isofields are of two types, **isofield of first kind**; wherein the isounit does not belong to the original field, and **isofield of second kind**; wherein the isounit belongs to the original field. The elements of the isofield are called as **isonumbers**. This leads to number of new terms and parallel developments of conventional mathematics.

7.2 Generalized Isostructures

- 'Isonumbers' is the generalization of conventional numbers formed by lifting conventional unit 1 to $\hat{1}$.
- In fact this lifting leads to a variety of algebraic structures which are often used in physics.
- The following flowchart is self explanatory.
  Isonumbers $\longrightarrow$ Isofields $\longrightarrow$ Isospaces $\longrightarrow$ Isotransformations $\longrightarrow$ Isoalgebras $\longrightarrow$ Isogroups $\longrightarrow$ Isosymmetries $\longrightarrow$ Isorepresentations $\longrightarrow$ Isogeometries etc.

8 The New structure

In view of the definition of an isofield [12], we can say that an isofield is an additive abelian group equipped with a new unit (called isounit) and isomultiplication defined appropriately so that the resulting structure becomes a field.
If the original field is alternative then the isofield also satisfies weaker *isoalternative laws* as follows. $\hat{a}\hat{\times}(\hat{b}\hat{\times}\hat{b}) = (\hat{a}\hat{\times}\hat{b})\hat{\times}\hat{b}$ **and** $\hat{a}\hat{\times}(\hat{a}\hat{\times}\hat{b}) = (\hat{a}\hat{\times}\hat{a})\hat{\times}\hat{b}$.
We mention two important proposition by Santilli.

Proposition 8.1. *The necessary and sufficient condition for the lifting (where the multiplication is lifted but elements are not)* $F(a,+,\times) \longrightarrow (\hat{F},+,\hat{\times}), \hat{\times} = \times T \times, \hat{1} = T^{-1}$ *to be an isotopy (that is for* $\hat{F}$ *to verify*

all axioms of the original field F) is that T is a non-null element of the original field F.

Proposition 8.2. *The lifting (where both the multiplication and the elements are lifted)*
$F(a,+,\times) \longrightarrow (\hat{F},+,\hat{\times}), \hat{a} = a \times \hat{1}, \hat{\times} = \times T \times, \hat{1} = T^{-1}$ constitutes an isotopy even when the multiplicative isounit $\hat{1}$ is not an element of the original field.

We propose three propositions which directly follow from the definition of isofield.

Proposition 8.3. *If $(F,+,\times)$ is a field and $(\hat{F},\hat{+},\hat{\times})$ is the corresponding isofield such that the isounit $\hat{I} \in F$ then $(F,+,\times) \cong (\hat{F},\hat{+},\hat{\times})$.*

Proposition 8.4. *If $(F,+,\times)$ is a field and $(\hat{F},\hat{+},\hat{\times})$ is the corresponding isofield such that the isounit $\hat{I} \notin F$ then $(F,+,\times)$ is isotopic to $(\hat{F},\hat{+},\hat{\times})$.*

Proposition 8.5. *Isofield corresponding to a non-commutative field is isotopic to the original field.*
The noncommutative ring of Quaternions is an example of this type.

8.1 Some Open Problems

- Can we construct finite isofields of first kind ?
- Can we construct finite isofields of second kind ?
- What is the structure of Quaternionic isofields ?
- Can a more generalized isofield be defined with a prescribed additive identity ?

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CONSEQUENCES OF SQUEEZING INITIALLY COHERENT (SEMI CLASSICAL) STATES WITH REGARDS TO FINDING IF THE COSMOLOGICAL CONSTANT IS OR IS NOT A 'VACUUM' FIELD

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Abstract

The analysis on this paper starts from an interesting question on the possibility that the cosmological constant should not be the vacuum field. This in response to a question from Professor. Karim . The consequences of that supposition are then analyzed in detail. for Earthbound experiments and also for CMBR data

Keywords: Vacuum energy, coherent, semi classical states, CMBR ,

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1 Introduction

Dr. Karim [38] mailed the author with the following question which will be put in quotes:

The challenge of resolving the following question: At the Big Bang the only form of energy released is in the form of geometry-– gravity. intense gravity field lifts vacuum fields to positive energies, So an electromagnetic vacuum of density 10^{122} kg/m3 should collapse under its own gravity. But this does not happen - that is one reason why the cosmological constant cannot be the vacuum field. Why?

Answering this question delves into what the initial state of the universe should be in terms of energy states outlined in this document. I.e. initially coherent states. That is the focal point of discussion of this article with an answer supplied by specific given examples and deduction from first principles of cosmology.

2. Semi classical models of entropy generation.

As was only recently seen, by works of 't Hooft[69,70] , and others, a positive cosmological constant has surprising consequences, such as a finite (not enormous) maximum entropy of the observable universe (see the holographic principle). We also note that Dyson, Kleban, and Susskind [25] in their 2002 article write the following

"The implication of such a description, as we have suggested in Section (1), is that Poincare recurrences are inevitable. Starting in a high entropy, "dead" configuration, if we wait long enough, a fluctuation will eventually occur in which the inflaton will wander up to the top of its potential, thus starting a cycle of inflation, re–heating, conventional cosmology and heat death. The frequency of such events is very low The typical time for (space–like boundary of de Sitter space) a fluctuation to occur is of order

$$T_r = \exp (S - S') \qquad (1)$$

where S is the equilibrium entropy and S′ is the entropy of the fluctuation. The fluctuations we have in mind correspond to early inflationary eras during which the entropy is probably of order 10^{10}, while the equilibrium entropy is of order 10^{120} . Thus

$$T_r = \exp 10^{120} \qquad (2)$$

This seems like an absurdly big time between interesting events, which by comparison last for a very short time. Nevertheless dismissing such long times as "unphysical" may be a symptom of extreme temporal provincialism."

The key point of their inquiry [25] is their statement, as given by

"Let us consider the entropy in observable matter in today's universe. It is of order 10^{100} This means that the number of microstates that are macroscopically indistinguishable from our world is exp (10^{100}). But only exp (10^{10}) of these states could have evolved from the low entropy initial state characterizing the usual inflationary starting point. The overwhelming majority of states which would have evolved into a world very similar to ours did not start in the usual low entropy ensemble."

I.e. how to make sense of a low entropy value of 10^{10} versus a present entropy of 10^{100}

Their question [25] ends up with

"To understand where they came from, imagine running these states backward in time until they thermalize in the eventual heat bath with entropy 10^{120}. Among the vast number exp (10^{120}) of possible initial starting points, a tiny fraction exp (10^{100}) will evolve into a world like ours. However, all but exp (10^{10}) of the corresponding trajectories (in phase space) are extremely unstable to tiny perturbations.

Having said that, the author wishes to come up with a model of how an initial cosmological constant could form which would be consistent with of 10^{10} counts of entropy

A way of doing so would be to require coherent states, initially very classical in nature, and to have deformation of these states in the initial states of the big bang.

Consider the following, namely, the brane world treatment of initial generation of entropy. Note that *no where in the brane theory model is there a description of how the instantons formed in the first place:*

Mathur's CQG article[50, 51] where he has a string winding interpretation of energy along the lines of putting as much energy E into string windings as possible via, $[n_1 + \bar{n}_1]LT = [2n_1]LT = E/2$, with T as tension, and where we are talking about n_1 wrappings of a string about a cycle of the torus , and $\bar{n}_1$ being 'wrappings the other way' , with the torus having a cycle of length L , which leads

to an entropy defined in terms of an energy value of , if mass $m_i = T_P \prod L_j$ (with T_P being the tension of the ith brane, and L_j being spatial dimensions of a complex torus structure) [50, 51]

$$E_{Total} = 2\sum_i m_i n_i \qquad (3)$$

This leads to entropy[47,48]

$$S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \qquad (4)$$

Our claim is that this very specific value of entropy for (4) above will in Planck interval of time at about the onset of inflation lead to[10,11]

$$\left\| \left[S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \right] \Big/ k_B \ln 2 \right| \approx [\#operations]^{3/4} \approx 10^8 - 10^{10} \qquad (5)$$

In getting such values, the question to ask is how the particular value of n_i were formed in the beginning

It is pretty obvious that Mathur is using N , as in the above equation , to signify a counting of would be branes,[50, 51] , in some sort of structure, in the initial beginning. If or not the structure is emergent is not stated clearly in his article. Now how about using energy, instead of branes , in terms of forming initial entropy. Such an approach allows us to use the 'cosmological "constant" parameter' approach directly.

3. Difficulty in visualizing what g_* is in the initial phases of inflation.

Secondly, we look for a way to link initial energy states, which may be pertinent to entropy, in a way which permits an increase in entropy from about 10^{10} at the start of the big bang to about 10^{100} today.

One such way to conflate entropy with an initial cosmological constant may be of some help, i.e. if $V_4|_{Threshold-volume-for-quantum-effects} \sim (10^{-4} cm)^3$ or smaller, i.e. in between the threshold value, and the cube of Planck length, one may be able to look at coming up with an initial value for a cosmological constant as given by Λ_{Max} as given by what Beckwith[10, 11] did with formulation from Padmanabhan [58,59]

$$\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 = E_{total} \tag{6}$$

Then making the following identification of total energy with entropy via looking at Λ_{Max} models, i.e. consider Park's model of a cosmological "constant" parameter scaled via background temperature[60]

$$\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}} \tag{7}$$

A linkage between energy and entropy, may be seen in the following construction, namely looking at what Kolb put in [40] , i.e.

$$\rho = \rho_{radiation} = (3/4)\cdot\left[\frac{45}{2\pi^2 g_*}\right]^{1/3}\cdot S^{4/3}\cdot r^{-4} \qquad (8)$$

Here, the idea would be, possibly to make the following equivalence, namely look at,

$$\left[\left[\frac{\Lambda_{Max} r^4}{8\pi G}\right]\cdot(4/3)\cdot\left[\frac{2\pi^2 g_*}{45}\right]^{1/3}\right]^{3/4} \sim S_{initial} \qquad (9)$$

Note that in the case that quantum effects become highly significant, that the contribution as given by $V_4|_{Threshold\text{-}volume\text{-}for\text{-}quantum\text{-}effects} \sim (10^{-4}cm)^3$ and potentially much smaller, as in the threshold of Plancks length, going down to possibly as low as 4.22419×10^{-105} m ^{3} = 4.22419×10^{-96} cm ^{3} leads us to conclude that even with very high temperatures, as an input into the initial entropy, that $S_{initial} \approx 10^{10}$ is very reasonable. Note though that Kolb and Turner [40] , however, have that g_* is at most about 120, whereas the author, in conversation with H. De La Vega, in 2009 [23] indicated that even the exotic theories of g_* have an upper limit of about 1200, and that it is difficult to visualize what g_* is in the initial phases of inflation. De La Vega stated in Como Italy, that he, as a conservative cosmologist, viewed defining g_* in the initial phases of inflation as impossible[23] .

One arguably needs a different venue as to how to produce entropy initially, and the way the author intends to present entropy, initially is through initial graviton production. The question of if gravitons, especially high frequency gravitons, can be detected will compose the last part of the manuscript. To start off with,

consider what if entropy were in a near 1-1 relations with , in initially very strongly curved space time with information. To do this , look at what was stated by Dyson, et al [25] , namely

3. The basic problem. Formation of 'particle' states in curved space time is difficult

To see this, consider an argument by Wald [76] which outlines the problem. Wald references a quantum field theory construction for bi linear mappings $\mu : \varsigma \times \varsigma \to \Re$, where ς is for a space of solutions to the Klein Gordon field, as defined via $\nabla^a \nabla_a \phi - m^2 \phi = 0$, and $\varsigma \cong \left\{ set\, \phi \right\}$ satisfying $\nabla^a \nabla_a \phi - m^2 \phi = 0$, for curved space time, and Wald specifically states there exists no **unique** positive definite $\mu : \varsigma \times \varsigma \to \Re$ for defining high frequency solutions in curved space time to define a space H , in terms of a complete basis set. The best one can do in curved space time, is to use a unitary equivalence of equivalent norms, to deal with the problems of curved space time, in the very early universe, i.e. as defined by Wald [76] , to have $C, C' > 0$ such that for all $\varsigma \cong \left\{ set\, \phi \right\}$ entries, that exist similar mappings μ_1, μ_2 for which

$$C\mu_1(\phi,\phi) \le \mu_2(\phi,\phi) \le C'\mu_1(\phi,\phi) \qquad (10)$$

I.e. to have mappings μ_1, μ_2 which give locally almost the same basis set with μ_1, μ_2 defined on neighboring curved space time geometry. Even when space is highly curved. there can be locally speaking relatively close basis sets formed via $\varsigma \cong \left\{ set\, \phi \right\}$, for almost the same μ_1, μ_2 . But in order to do this, one would need

to have $\nabla^a \nabla_a \phi - m^2 \phi = 0$ obeyed, with $\varsigma \cong \left\{ set\, \phi \right\}$, and $\mu : \varsigma \times \varsigma \to \Re$, which places a premium upon having semi classical solutions, i.e. if one goes to purely quantum descriptions of entries for $\varsigma \cong \left\{ set\, \phi \right\}$ one will have difficulty forming the space H , in terms of a complete basis set in the sense specified by Wald [76]. This leads us to the question of how to obtain solutions which are congruent to semi classical approximations, for the evolution of space time as we know it. I.e. our preferred procedure is to use coherent states, in terms of initial $\varsigma \cong \left\{ set\, \phi \right\}$, so as to get about the datum, as mentioned by Wald that the use of (10) in curved space time makes forming an initial particle in space time geometry very difficult to do. The best which could be done would be to make the functions μ_1, μ_2 as 'close to one another as possible' . So then what can be said is how to do that, and to make the process linked to structure formation later on.

4 Difficulties in obtaining a mostly classical treatment of $\varsigma \cong \left\{ set\, \phi \right\}$ initially speaking

Aside from the conundrum of squeezed states, which will be addressed in the next section, one of the issues to consider is what to do with the mix of DM states. As mentioned in the following . When DE becomes important, the density of DE contribution goes as [58,59]

$$\rho_{VAC} \sim \frac{\Lambda_{observed}}{8\pi G} \sim \sqrt{\rho_{UV} \cdot \rho_{IR}} \sim \sqrt{l_{Planck}^{-4} \cdot l_H^{-4}} \sim l_{Planck}^{-2} \cdot H_{observed}^2 \tag{11}$$

$$\Delta\rho \approx \text{a dark energy density} \sim H^2_{observed}/G \qquad (12)$$

This is, however, when DE is important, and that at best, we do not consider DE in the initial beginning of the universe. I.e. in the beginning the following mix of initial states is probably correct

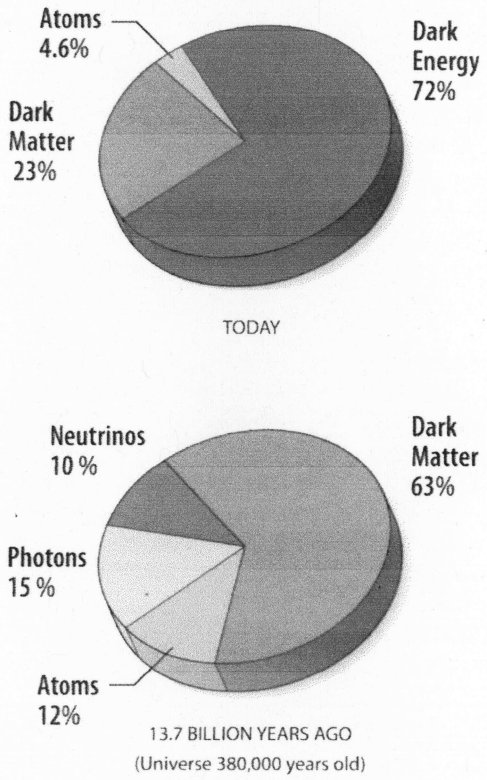

Figure 1: from G. Hingsaw presentation, in COMO, Italy, July 2009 at the ISAPP [35]

How DM and other constituent parts of the early 380 thousand year old universe evolved to have connections with KK gravitons is connected closely with the following

The emphasis is placed on DM, as opposed to DE, and this means, among other things, considering different mixtures of DM to consider as contributing to space time evolution. i.e. what can be said about a power spectra[10,11]

$$P \sim \frac{1}{150\pi^2} \cdot \frac{V(\phi)}{\epsilon} \qquad (13)$$

This is assuming a slow roll parameter treatment with $\epsilon << 1$, and for $t > t_P$.. (13) would be growing fairly rapidly in line with what is said about (13) above. An increase in scalar power, is then proportional to an increase in entropy[10,11]

$$\left|\frac{\Delta E}{l_P^3}\right| \sim \left|\frac{\Delta P \epsilon 150\pi^2}{l_P^3}\right| \approx |\Delta S| \qquad (14)$$

The issue of (14) would be in its own way resolvable if one is consider a way to differentiate different values of DM, in a contribution to the term ΔE , which could reflect themselves in the delineation of a density contribution to ΔE along the lines of Boyarsky, Lesgourgues, Ruchayskiy and Matteo Viel(2009) [17] have strict Baysian s tatistical limits as to what sort of warm to cold dark matter mixes are allowed. A mix of warm and cold DM mixes would allow for . Here, we can assume that $\Delta E / l_P^3 \sim \delta\rho$

A worthy exercise is to consider if $\Delta E / l_P^3 \sim \delta\rho$ can reflect DM warm and cold candidates, and to what degree this would reflect how the cosmological "constant" would come about. Begin by considering the possible identification,

namely if H_{early} is the initial behavior of the Hubble parameter, and one starts with

$$\frac{\left[\Delta E/l_P^3\right]}{\left[H_{early}^2/l_P^2\right]} \approx \frac{\Delta S}{\left[H_{early}^2/l_P^2\right]} \sim \frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \sim \frac{\delta\rho}{\rho} \qquad (15)$$

where we will put in a candidate for the ΔS for initial conditions, and then use that as far as answering questions as far as formulating an answer as far as entropy fluctuations, and candidates for density fluctuations, as well as early values of the Hubble parameter. One of their basic result, which is put here, $\rho_{Baryons}, \rho_{Cold-Dark-Matter}, \rho_{Warm-Dark-Matter}$ refer to density profiles, of the respective baryons, CDM, and WDM candidates, whereas, the density fluctuations $\delta_{Baryons}, \delta_{Cold-Dark-Matter}, \delta_{Warm-Dark-Matter}$ are with regards to the fluctuations of these density values. So [17]

$$\left(\frac{\delta\rho}{\rho}\right) \equiv \frac{\rho_{Baryons}\delta_{Baryons} + \rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}}{\rho_{Baryons} + \rho_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}} \qquad (16)$$

In rough details, if axions are CDM, and KK gravitons are for WDM, then up to a point, $\rho_{Warm-Dark-Matter}$ would dominate (14) in earlier times, ie. Up to Z~1100. However, Boyarsky, et al [17] (2009) also stress that as of the recent era, i.e. probably for Z~.55 to Z~0 today , they would expect to see the following limiting behavior

$$\begin{aligned} &\delta_{Baryons} \equiv \delta_{CDM}, \\ &\delta_{WDM} << \delta_{CDM} \end{aligned} \qquad (17)$$

In earlier times, what is put in, with regards to (17) would be probably far different . However, up in the present era, the denominator of (16) would be dominated by KK DM, whereas there would be rough equality in the

contributions $\rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter}, \rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}$, with the baryon contribution to the numerator being ignorable, due to how small baryon values would be for Z~.55 to Z~0 today. Somehow, contributions as to (15) should be compared with.

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \frac{k^{3/2}|\delta_k|}{\sqrt{2\pi}} \propto \frac{k^{(3/2)+3\alpha-3/2}}{\sqrt{2\pi}} \approx \left(1/\sqrt{2\pi}\right)\cdot k^{3\alpha} \qquad (18)$$

, where $-.1 < \alpha < 0.2$, and $\alpha \equiv 0 \Leftrightarrow n_s \equiv 1$ and to first order, $k \cong Ha$. following limits as to what is picked as of (15) in early and later times should be reconciled with.

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \left(1/\sqrt{2\pi}\right)\cdot k^{3\alpha} \sim \frac{H^2}{\dot{\phi}} \propto 10^{-4}-10^{-5} \qquad (19)$$

Having such a relatively small value of $l_P^2 \propto \left[1.616\times10^{-35} meters\right]^2$ as placed with $\Delta S \sim 10^{10}$

$$10^{-4}-10^{-5} \sim \frac{l_P^2\cdot\Delta S}{H_{Early}^2} \qquad (20)$$

This will lead to comparatively low values for H_{Early}^2 which will be linked to the behavior of a cosmological 'constant' parameter value, which subsequently changes in value later. I.e., (20) will be for a configuration just before the onset of the big bang itself, which will have implications for Dr. Karims [38] question. .Assuming that one has a particle count, which will be how $\Delta S \sim 10^{10}$ is initially formulated, will lead to questions in how the cosmological constant evolves, and reaches its present value .Note that if H_{Early}^2 is picked as initially small, it becomes enormous when $\Delta S \sim 10^{10}$ pushes to value a bit later to approaching

much later time values of 10^{100}. This shift in value for H^2_{Early}, from a small to a much later value later on, will go to the main issue of initially , with $l_P^2 \propto \left[1.616\times10^{-35} meters\right]^2$

$$H^2_{Early} \sim \left[\Lambda_{Cosmological} \cdot l_P^2 / 8\pi G\right] \tag{21}$$

And, also,

$$\frac{l_P^2 \cdot \Delta S}{H^2_{Early}} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{Cosmological}} \sim 10^{-4} - 10^{-5} \tag{22}$$

An initially small value for $\Lambda_{Cosmological}$, in line with $\Delta S \sim 10^{10}$ just at the onset of inflation, plus increases of $\Lambda_{Cosmological}$ as $\left[\Delta S \sim 10^{10}\right]_{initial} \rightarrow 10^{58}$ at the end of inflation and about the start of the radiation era corresponds to initially un squeezed initial states which become squeezed coherent states . The discussion of coherent squeezed states comes next.

5. How squeezed state conditions at the onset of inflation affects usual attempts at measurement of coherent relic graviton states.

Now what could be said about forming states close to classical representations of gravitons? Venkatartnam, and Suresh, [73] 2007 built up a coherent state via use of a displacement operator $D(\alpha) \equiv \exp\left(\alpha \cdot a^+ - \alpha^* \cdot a\right)$, applied to a vacuum state, where α is a complex number, and a, a^+ as annihilation, and creation operations $\left[a, a^+\right] = 1$, where one has

$$|\alpha\rangle = D(\alpha) \cdot |0\rangle \tag{23}$$

It is appropriate to call the initial vacuum state as initially coherent, and not necessarily formed via purely quantum processes. The vacuum state, $|0\rangle$ may be explained as stated by Bellini [12], with the initial vacuum state as described by using the initial time to be the Planckian time $G^{1/2}$. Later, the universe may undergo a second-order phase transition. A five dimensional vacuum state may or may not have a quantum genesis. For the sake of argument, consider if $|0\rangle$ arises as due to what Volovik [75] brought up in. an initial state $|0\rangle$, would be consistent with stating that " *its vacuum energy density is exactly zero without fine tuning, if: there are no external forces acting on the liquid; there are no quasiparticles which serve as matter; no space-time curvature; and no boundaries which give rise to the Casimir effect. Each of these four factors perturbs the vacuum state and induces the nonzero value of the vacuum energy density of order of energy density of the perturbation. This is the reason, why one must expect that in each epoch the vacuum energy density is of order of matter density of the Universe, or/and of its curvature, or/and of the energy density of smooth component*"

However, what one sees in string theory, is a situation where a vacuum state as a template for graviton nucleation is built out of an initial vacuum state, $|0\rangle$. To do this though, as Venkatartnam, and Suresh did [73] , involved using a squeezing operator $Z[r,\vartheta]$ [66] defining via use of a squeezing parameter r as a strength of squeezing interaction term , with $0 \le r \le \infty$, and also an angle of squeezing, $-\pi \le \vartheta \le \pi$ as used in $Z[r,\vartheta] = \exp\left[\frac{r}{2}\cdot\left([\exp(-i\vartheta)]\cdot a^2 - [\exp(i\vartheta)]\cdot a^{+2}\right)\right]$, where combining the $Z[r,\vartheta]$ with (24) leads to a single mode squeezed coherent state, as they define it via

$$|\varsigma\rangle = Z[r,\vartheta]|\alpha\rangle = Z[r,\vartheta]D(\alpha)\cdot|0\rangle \xrightarrow[\alpha\to 0]{} Z[r,\vartheta]\cdot|0\rangle \qquad (24)$$

The right hand side. of (24) given above becomes a highly non classical operator, i.e. in the limit that the super position of states $|\varsigma\rangle \xrightarrow[\alpha\to 0]{} Z[r,\vartheta]\cdot|0\rangle$ occurs, there is a many particle version of a 'vacuum state' which has highly non classical properties. Squeezed states, for what it is worth, are thought to occur at the onset of vacuum nucleation, but what is noted for $|\varsigma\rangle \xrightarrow[\alpha\to 0]{} Z[r,\vartheta]\cdot|0\rangle$ being a super position of vacuum states, means that classical analog is extremely difficult to recover in the case of squeezing, and general non classical behavior of squeezed states. Can one, in any case, faced with $|\alpha\rangle = D(\alpha)\cdot|0\rangle \neq Z[r,\vartheta]\cdot|0\rangle$ do a better job of constructing coherent graviton states, in relic conditions, which may not involve squeezing ?. Note L. Grishchuk [31] wrote in (1989) in "On the quantum state of relic gravitons", where he claimed in his abstract that 'It is shown that relic gravitons created from zero-point quantum fluctuations in the course of cosmological expansion should now exist in the squeezed quantum state. The authors have determined the parameters of the squeezed state generated in a simple cosmological model which includes a stage of inflationary expansion. It is pointed out that, in principle, these parameters can be measured experimentally'. Grishchuk, et al, [31] (1989) reference their version of a cosmological perturbation h_{nlm} via the following argument. How we work with the argument will affect what is said about the necessity, or lack of, of squeezed states in early universe cosmology. From [31] (1989),, where h_{nlm} has a component $\mu_{nlm}(\eta)$ obeying a parametric oscillator equation, where K is a measure of curvature which is $=\pm 1,0$, $a(\eta)$ is a scale factor of a FRW metric, and $n = 2\pi\cdot[a(\eta)/\lambda]$ is a way to scale a wavelength, λ, with n, and with $a(\eta)$

$$h_{nlm} \equiv \frac{l_{Planck}}{a(\eta)} \cdot \mu_{nlm}(\eta) \cdot G_{nlm}(x) \tag{25}$$

$$\mu''_{nlm}(\eta) + \left(n^2 - K - \frac{a''}{a} \right) \cdot \mu_{nlm}(\eta) \equiv 0 \tag{26}$$

If $y(\eta) = \frac{\mu(\eta)}{a(\eta)}$ is picked, and a Schrodinger equation is made out of the Lagrangian [31] used to formulate (26) above, with $\hat{P}_y = \frac{-i}{\partial y}$, and $M = a^3(\eta)$, $\Omega = \frac{\sqrt{n^2 - K^2}}{a(\eta)}$, $\breve{a} = [a(\eta)/l_{Planck}] \cdot \sigma$, and $F(\eta)$ an arbitrary function. $y' = \partial y / \partial \eta$.

Also, we have a finite volume $V_{finite} = \int \sqrt{^{(3)}g} d^3x$

Then the Lagrangian for deriving (26) is (and leads to a Hamiltonian which can be **also** derived from the Wheeler De Witt equation), with $\varsigma = 1$ for zero point subtraction of energy [31]

$$L = \frac{M \cdot y'^2}{2a(\eta)} - \frac{M^2 \cdot \Omega^2 a \cdot y^2}{2} + a \cdot F(\eta) \tag{27}$$

$$\frac{-1}{i} \cdot \frac{\partial \psi}{a \cdot \partial \eta} \equiv \hat{H}\psi \equiv \left[\frac{\hat{P}^2{}_y}{2M} + \frac{1}{2} \cdot M\Omega^2 \hat{y}^2 - \frac{1}{2} \cdot \varsigma \cdot \Omega \right] \cdot \psi \tag{28}$$

Then there are two possible solutions to the S.E. Grushchuk [31] created in 1989, one a non squeezed state, and another a squeezed state[31] . So in general we work with

$$y(\eta) = \frac{\mu(\eta)}{a(\eta)} \equiv C(\eta) \cdot \exp(-B \cdot y) \tag{29}$$

The non squeezed state has a parameter $B|_{\eta} \xrightarrow[\eta \to \eta_b]{} B(\eta_b) \equiv \omega_b/2$ where η_b is an initial time, for which the Hamiltonian given in (28) in terms of raising/ lowering operators is 'diagonal', and then the rest of the time for $\eta \neq \eta_b$, the squeezed state for $y(\eta)$ is given via a parameter B for squeezing which when looking at a squeeze parameter r, for which $0 \leq r \leq \infty$, then (10) has, instead of $B(\eta_b) \equiv \omega_b/2$ [31]

$$B|_{\eta} \xrightarrow[\eta \neq \eta_b]{} B(\omega, \eta \neq \eta_b) \equiv \frac{i}{2} \cdot \frac{(\mu/a(\eta))'}{(\mu/a(\eta))} \equiv \frac{\omega}{2} \cdot \frac{\cosh r + [\exp(2i\vartheta)] \cdot \sinh r}{\cosh r - [\exp(2i\vartheta)] \cdot \sinh r} \quad (30)$$

Taking Grishchuck's [31] formalism literally, a state for a graviton/ GW is not affected by squeezing when we are looking at an initial frequency, so that $\omega \equiv \omega_b$ initially corresponds to a non squeezed state which may have coherence, but then right afterwards, if $\omega \neq \omega_b$ which appears to occur whenever the time evolution, $\eta \neq \eta_b \Rightarrow \omega \neq \omega_b \Rightarrow B(\omega, \eta \neq \eta_b) \equiv \frac{i}{2} \cdot \frac{(\mu/a(\eta))'}{(\mu/a(\eta))} \neq \frac{\omega_b}{2}$ A reasonable research task would be to determine, whether or not $B(\omega, \eta \neq \eta_b) \neq \frac{\omega_b}{2}$ would correspond to a vacuum state being initially formed right after the point of nucleation, with $\omega \equiv \omega_b$ at time $\eta \equiv \eta_b$ with an initial cosmological time some order of magnitude of a Planck interval of time $t \approx t_{Planck} \propto 10^{-44}$ seconds The next section will be to answer whether or not there could be a point of no squeezing, as Grishchuck implied, for initial times [31] , and initial frequencies, and an immediate transition to times, and frequencies afterwards, where squeezing was mandatory. Note that in 1993, Grischchuk [32] further extended his analysis, with respect to the same point of departure, i.e. what to do with when $|\alpha\rangle = D(\alpha) \cdot |0\rangle \neq Z[r, \vartheta] \cdot |0\rangle$. Having $|\alpha\rangle = D(\alpha) \cdot |0\rangle$ with $D(\alpha)$ a possible

displacement operator, seems to be in common with $B(\eta_b) \equiv \omega_b/2$, whereas $|\alpha\rangle = Z[r,\vartheta]\cdot|0\rangle$ which is highly non classical seems to be in common with a solution for which $B(\omega_b) \neq (\omega_b/2)$ This leads us to the next section, i.e. does $B(\eta_b) \equiv \omega_b/2$ when of time $t \approx t_{Planck} \propto 10^{-44}$ seconds, and then what are the initial conditions for forming 'frequency' $\omega \equiv \dot{\omega}_b$?

6 Necessary & sufficient conditions for String/Brane theory graviton coherent states?

A curved space-time is a coherent background of gravitons, and therefore in string theory is a coherent state Joseph Gerard Polchinski [61] starting with the typical small deviation from flat space times as can be written up by $G_{uv}(X) = \eta_{uv} + h_{uv}(X)$, with η_{uv} flat space time, and the Polyakov action, is generalized as follows, the S_σ Polyakov action is computed and compared with exponentiated values

$$S_\sigma = \frac{1}{4\pi\alpha'}\cdot\int_M d^2\sigma\cdot\sqrt{g}\cdot g^{ab}\cdot G_{uv}(X)\cdot\partial_a X^\mu\partial_b X^\nu \qquad (31)$$

Becomes

$$\exp(-S_\sigma) = [\exp(-S_P)]\cdot\left[1-\frac{1}{4\pi\alpha'}\cdot\int_M d^2\sigma\cdot\sqrt{g}\cdot g^{ab}\cdot h_{uv}(X)\cdot\partial_a X^\mu\partial_b X^\nu + \ldots\right] \qquad (32)$$

Polochinski [61] writes that the term of order h is the vertex operator for the graviton state of the string, with $h_{uv}(X) \equiv -4\pi g_c\cdot\exp\left[-ikX_{S_{uv}}\right]$, and the action of S_σ a coherent state of a graviton. Now the important question to ask is if this coherent state of a graviton, as mentioned by Polochinski can hold up in relic,

early universe conditions. Dick[24] , in 2001, argued as stating that the "graviton multiple as one particular dark matter source in heterotic string theory. In particular, it is pointed out that an appreciable fraction of dark matter from the graviton multiplet requires a mass generating phase transition around $T_c \simeq 10^8$ GeV, where the symmetry partners of the graviton would evolve from an ultra hard fluid to pressure less dark matter. Indicates $m \simeq 10$ MeV for the massive components of the graviton multiplet". This has a counter part in a presentation made by Berkenstein [13] (2004) with regards to BPS states, and SHO models for $AdS_5 \times S^5$ geometry. The upshot is that string theory appears to construct coherent graviton states, but it has no answer to the problem that Ford [26] (1995), and note that Grishchuck [31,32,33] , wrote on if the existing graviton coherent states would be squeezed into non classical configurations in relic conditions.

7 A suggestion from the author as to initial graviton production, and its links to $|0\rangle$

Beckwith[10, 11] has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant [45] , as given by

$$\tilde{\alpha} \equiv e^2/\hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\tilde{\lambda}}{hc} \tag{33}$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be

put below, namely: Argue that when taking the log, that the 1/N term drops out. As used by Ng [55] there is a partition function we call

$$Z_N \sim (1/N!)\cdot\left(V/\bar{\lambda}^3\right)^N \tag{34}$$

This, according to Ng, [56] leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith [10,11]

$$S \approx N\cdot\left(\log\left[V/\lambda^3\right]+5/2\right)\approx N \tag{35}$$

Eventually, the author hopes to put on a sound foundation what 'tHooft is doing with respect to t'Hooft [69,70] deterministic quantum mechanics [69,70] and equivalence classes embedding quantum particle structures. If one uses the wave functional

$$\Psi_{i,f}\left[\phi(\mathbf{x})\right]\Big|_{\phi=\phi_{cl,f}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\,\alpha\left[\phi_{Ci,f}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\} \tag{36}$$

With $\phi_0(x)$ being equivalence classes to fit in a kink anti kink structure with t'Hooft's work [69,70] and tied it in with equivalence classes, and mixed it in with a kink anti kink structure given by the following figures from Beckwith's dissertation [8] . The first one is involving the use of instantons and what is known as domain wall approximations. Fig 2. below represents how a Cooper pair charge can be used to ascertain an instanton- anti instanton structure would be organized as of CDW, for quasi one dimensions. The second, Fig 3 is how an equivalence class structure could be put in, and what the consequences would be. I.e. as given by Beckwith [8]

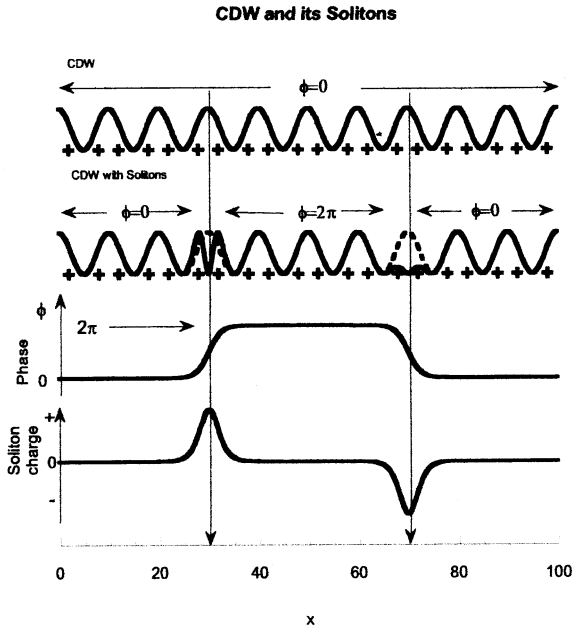

Fig 2, **The pop up effects of an intanton-anti-instanton in Euclidian space** [10,11]

Doing so will answer questions raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere expect in a localized "bump" of curvature. Furthermore, making a count of gravitons with $S \approx N \sim 10^{20}$ gravitons, with use of the formula from Lloyd [46] , of $I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} \sim 10^{20}$ as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced

graviton[3]. What the author, Beckwith, sees is that since instanton- anti instanton pairs does not have to travel slowly, as has been proved by authors in the 1980s that gravitons if nucleated in a fashion as indicated by Fig. 3 will not be influenced by differences in three or more dimensional geometry from an emergent structure, and as indicated by Isbanez and Verdaguer [36] The instanton – anti instanton structure allows for rapid travel This is in tandem with findings with results obtained by Miller [53]..

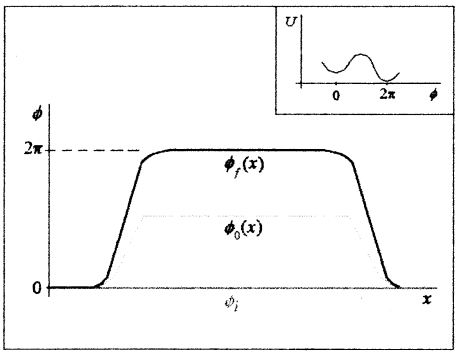

Fig. 3: The pop up effects of an intanton-anti-instanton in Euclidian space [8]

Also, an instanton - anti instanton structure may allow us to be able to answer the following. The stretch-out of a graviton wave, greater than the size of the solar system, gives, an upper limit of a graviton mass due to wave length $\lambda_{graviton} > 300 \cdot h_0 kpc \Leftrightarrow m_{graviton} < 2\times 10^{-29} h_0^{-1} eV$ [3]. I. e. stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful limits due to experimental searches, as presented by Buonanno [19] have narrowed the upper limit to $10^{-20} h_0^{-1} eV$. An instanton – anti instanton structure to the graviton, if confirmed, plus experimental confirmation of mass, plus perhaps $n \sim 10^{20}$ gravitons $\approx 10^{20}$ entropy counts, implies up to $\approx 10^{27}$ operations. If so, there is a one-to-one relationship between an operation and a bit of information, so a graviton has at least one bit of information. And that may be enough to determine the conditions needed to determine if parameter inputs gives information and structure from a prior universe to our present cosmos. Finally, the datum referred as combined with $S \approx N$ as a way to relate the graviton count with entropy may be a way to make inter connection between the inflaton picture of entropy generation and entropy connected/ generated with a numerical count of gravitons. The author contends that the above

formalism for a graviton as an emergent particle, with a slight mass in four dimensions is consistent with what Sahni and Habib [65] worked with, in 1998. Experimental verification of this would be important for determining if or not theories purporting to show increasing or decreasing values of the gravitational constant were valid, e.g. of the sort given by Singh [66] are based upon firm experimental foundations.

8 Does LQG give us more direct arguments as to coherent states, squeezed states, and the break down of classical behavior at the onset of inflation?

Rovelli, in 2006 [64], states that a vertex amplitude that contributes to a coherent graviton state is the exponential of the Regge action: the other terms, that have raised doubts on the physical viability of the model, are suppressed by the phase of the vacuum state, and Rovelli [64]writes a coherent vacuum state as given by a Gaussian peaked on parts of the boundary Σ_d of a four dimensional sphere.

$$\Psi_q[s] = \Psi_q(\Gamma, j_{m,n}) \qquad (37)$$

Rovelli [64] states that "bad" contributions to the behavior of. (37) are cancelled out by an appropriate (Gaussian?) vacuum wave functional which has 'appropriately' chosen contributions from the boundary Σ_d of a four dimensional sphere. This is to avoid trouble with "bad terms" from what is known as the Barret – Crane vertex amplitude contributions, which are can be iminized by an appropriate choice of vacuum state amplitude being picked. Rovelli calculated some components of the graviton two-point function and found that the Barrett-Crane vertex yields a wrong long-distance limit. A problem, as stated by Lubos Motel [54] (2007), that there are infinitely many other components of the correlators in the LQG that are guaranteed not to work unless an infinite number of adjustments are made. The criticism is harsh, but until one really knows admissible early universe geometry one cannot rule out the Rovelli approach, or confirm it. In addition, Jakub Mielczarek [52] (2009)

considered tensor perturbations produced at a bounce phase in presence of the holonomy corrections. Here bounce phase and holonomy corrections originate from Loop Quantum Cosmology What comes to the fore are corrections due to what is called quantum holonomy, l.. A comment about the quantum bounce. i.e. what is given by Chiou and Li,[20] (2009) is that there is a branch match up between a prior to a present set of Wheeler De Witt equations for a prior to present universe, as far as modeling how the quantum bounce links the two Wheeler De Witt solution branches, i.e. one Wheeler De Witt wave function for a prior universe, and another wave function for a present universe. Furthermore, Ashtekar [3, 4] (2006) wrote a simple treatment of the Bounce causing Wheeler De Witt equation along the lines of, for $\rho_* \approx const \cdot (1/8\pi G\Delta)$ as a critical density, and Δ the eignvalue of a minimum area operator. Small values of Δ imply that gravity is a repulsive force, leading to a bounce effect.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{8\pi G}{3} \cdot \rho \cdot (1 - (\rho/\rho_*)) + H.O.T. \qquad (38)$$

Furthermore, Bojowald [14,15,16] (2008) specified criteria as to how to use an updated version of Δ and $\rho_* \approx const \cdot (1/8\pi G\Delta)$ in his GRG manuscript on what could constitute grounds for the existence of generalized squeezed initial (graviton?) states. Bojowald[14,15,16] (2008) was referring to the existence of squeezed states, as either being necessarily, or NOT necessarily a consequence of the quantum bounce. As Bojowald [14,15,16] (2008) wrote it up, in both his equation (26) which has a quantum Hamiltonian $\langle \hat{V} \rangle \approx H$, with

$$\left.\frac{d\langle \hat{V} \rangle}{d\phi}\right|_{\phi \approx 0} \xrightarrow[existence-of-un-squeezed-states \Leftrightarrow \phi \approx 0]{} 0 \qquad (39)$$

, and $\hat{V}$ is a 'volume' operator where the ' volume' is set as V , Note also, that Bojowald [14,15,16] has , in his initial Friedman equation, density values $\rho \equiv \frac{H_{matter}(a)}{a^3}$, so that when the Friedman equation is quantized, with an initial internal time given by ϕ , with ϕ becoming a more general evolution of state variable than 'internal time'. If so, Bojowald[14,15,16] (2008) writes, when there are squeezed states

$$\left.\frac{d\langle \hat{V} \rangle}{d\phi}\right|_{\phi \neq 0} \xrightarrow[existence-of-squeezed-states]{} N(value) \neq 0 \qquad (40)$$

For (40), which is incidentally when links to classical behavior breaks down, and when the bounce from a universe contracting goes to an expanding present universe. Bojowald also writes that if one is looking at an isotropic universe, that as the large matter 'H' increases, that in certain cases, one observes more classical behavior, and a reduction in the strength of a quantum bounce. Bojowald [14,15,16] states that "Especially the role of squeezed states is highlighted. The presence of a bounce is proven for uncorrelated states, but as squeezing is a dynamical property and may change in time" The upshot is that although it is likely in a quantum bounce state that the states should be squeezed, it is not a pre requisite for the states to always start off as being squeezed states. .So a physics researcher can ,look at if an embedding of the present universe in a higher dimensional structure which could have lead to a worm hole from a prior universe to our présent for re introduction of inflationary growth

9 .Other models. Do worm hole bridges between different universes allow for initial un squeezed states? Wheeler De Witt solution with pseudo time component added in.

This discussion is to present a not so well known but useful derivation of how instanton structure from a prior universe may be transferred from a prior to the present universe.

1. The solution as taken from L. Crowell's [21] ([2005) book, and re produced here, as referenced by Beckwith (2008,2009) has many similarities with the WKB method. I.e. it is semi CLASSICAL.
2. left unsaid is what embedding structure is assumed
3. A final exercise for the reader. Would a WKB style solution as far as transfer of 'material' from a prior to a present universe constitute procedural injection of non compressed states from a prior to a present universe? Also if uncompressed, coherent states are possible, how long would they last in introduction to a new universe?

This is the Wheeler-De-Witt equation with pseudo time component added. From Crowell[21]

$$-\frac{1}{\eta r}\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{\eta r^2}\cdot\frac{\partial\Psi}{\partial r}+rR^{(3)}\Psi=\left(r\eta\phi-r\ddot{\phi}\right)\cdot\Psi \qquad (41)$$

This has when we do it $\phi\approx\cos(\omega\cdot t)$, and frequently $R^{(3)}\approx$ constant, so then we can consider

$$\phi\cong\int_0^\infty d\omega\left[a(\omega)\cdot e^{ik_\omega x^\mu}-a^+(\omega)\cdot e^{-ik_\omega x^\mu}\right] \qquad (42)$$

In order to do this, we can write out the following for the solutions to (42) above.

$$C_1 = \eta^2 \cdot \left(4 \cdot \sqrt{\pi} \cdot \frac{t}{2\omega^5} \cdot J_1(\omega \cdot r) + \frac{4}{\omega^5} \cdot \sin(\omega \cdot r) + (\omega \cdot r) \cdot \cos(\omega \cdot r) \right) \quad (43)$$
$$+ \frac{15}{\omega^5} \cos(\omega \cdot r) - \frac{6}{\omega^5} Si(\omega \cdot r)$$

And

$$C_2 = \frac{3}{2 \cdot \omega^4} \cdot (1 - \cos(\omega \cdot r)) - 4e^{-\omega \cdot r} + \frac{6}{\omega^4} \cdot Ci(\omega \cdot r) \quad (44)$$

This is where $Si(\omega \cdot r)$ and $Ci(\omega \cdot r)$ refer to integrals of the form $\int_{-\infty}^{x} \frac{\sin(x')}{x'} dx'$ and $\int_{-\infty}^{x} \frac{\cos(x')}{x'} dx'$. . Next, we should consider whether or not the instanton so formed is stable under evolution of space-time leading up to inflation. To model this, we use results from Crowell [21] (2005) on quantum fluctuations in space-time, which gives a model from a pseudo time component version of the Wheeler-De-Witt equation, with use of the Reinssner-Nordstrom metric to help us obtain a solution that passes through a thin shell separating two space-times. The radius of the shell $r_0(t)$ separating the two space-times is of length l_P in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom
metric

$$dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \quad (45)$$

This has:

$$F(r)=1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{\Lambda}{3}\cdot r^2 \xrightarrow[T\to 10^{32}\,Kelvin\sim\infty]{} -\frac{\Lambda}{3}\cdot(r=l_P)^2 \quad (46)$$

This assumes that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park[60] (2003), leading to

$$\frac{\partial F}{\partial r}\sim -2\cdot\frac{\Lambda}{3}\cdot(r\approx l_P)\equiv \eta(T)\cdot(r\approx l_P) \quad (47)$$

As a wave functional solution to a Wheeler-De-Witt equation bridging two space-times, similar to two space-times with "instantaneous" transfer of thermal heat, as given by Crowell[21] (2005)

$$\Psi(T)\propto -A\cdot\{\eta^2\cdot C_1\}+A\cdot\eta\cdot\omega^2\cdot C_2 \quad (48)$$

This has $C_1=C_1(\omega,t,r)$ as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function. This also applies to the second cyclical wave function $C_2=C_2(\omega,t,r)$, where the values
$C_1=(43)$ and $C_2=$ (44) Here, (48) is a solution to the pseudo time WDM equation. The question which will be investigated is if (48) is a way to present either a squeezed or un squeezed state.. Note that the Crowell construction of a worm hole bridge is in some ways similar to of use of conjugate momentum π^{ij} of h_{ij} generalized momentum variables, also known as conjugate momentum $\hat{\pi}^{ij}\equiv\frac{\hbar}{i}\cdot\frac{\partial}{\partial\cdot h_{ij}}$, leading to the sort of formalism as attributed to the (1991) article , of [27]

$$\Psi(h_{ij})\approx\left|\exp\int d^3x\cdot\pi^{ij}\cdot h_{ij}\right|_T \quad (49)$$

Now in the case of what can be done with the worm hole used by Crowell, with, if $\hbar \equiv 1$, $\hat{\pi}^{ij} \equiv -i\frac{\delta}{\delta \cdot g_{ij}}$, $\hat{\pi}^{\theta\theta} \equiv -\frac{i}{2r}\frac{\partial}{\partial \cdot r}$, $\hat{\pi}^{tt} \equiv -i \cdot \left(\frac{\partial F(r)}{\partial r}\right)^{-1} \cdot \frac{\partial}{\partial \cdot r}$, and a kinetic energy value as given of the form $\hat{\pi}^{\theta\theta}\hat{\pi}^{tt} + \hat{\pi}^{tt}\hat{\pi}^{\theta\theta}$. The supposition which we have the worm hole wave functional may be like, so, use the wave functional looking like $\Psi(g_{ij}) \approx \left[\exp \int d^3x \cdot [\pi^{ij}] \cdot g_{ij}\right]_T$ where the g_{ij} for the Weiner-Nordstrom metric will be

$$dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \equiv g^{ij}dx^i dx^j \equiv \frac{\Lambda}{3} \cdot (r = l_P)^2 \cdot dt^2 - \frac{dr^2}{\frac{\Lambda}{3} \cdot (r = l_P)^2} + d\Omega^2 \quad (50)$$

The main problem in these assumptions about how likely one can measure GW at all is in the assumed impossibility of measuring a 'strain factor' $h << 10^{-21}$ According to Li, et al [43] (2009), $h \sim 10^{-30}$ is the sensitivity factor needed to measure GW. Weiss, in personal communications [77] (2009) states flatly in personal communications with the author that measurements of $h \sim 10^{-30}$ are impossible with currently achievable GW technology. To answer this, the author states that there does exist an argument by Dr. Fangyu Li's personal notes [43] and personal communications (2009), which implies that relic GW, and by implicit assumption, gravitons, are not to be ruled out as Weiss stated was the case in personal communications with the author The assumptions the author is making is that with with careful calibration, there is a way to obtain measurable relic GW, and also, possibly, graviton measurements. The author wishes to thank Professor Rainer Weiss, of MIT, in ADM 50 [77] , in November 7th (2009) for explaining the implications of a formula for HFGW of at least 1000 Hertz for GW which is a start in the right direction i.e., a strain value of, if L is the Interferometer length, and N is the number of quanta / second at a beam splitter, and τ is the integration time.

$$h \sim \frac{\lambda}{Lb\sqrt{N\tau}} \tag{51}$$

For LIGO systems, and their derivatives, the usual statistics and technologies of present lasers as bench marked by available steady laser in puts given by Eq (1.50) appear to limit $h \sim 10^{-23}$. The problem is that as Weiss [77] explained to the author, one of the most active, and perhaps guaranteed to obtain GW sources involves the interaction of super massive black holes in the center of colliding galaxies, which would need $h \sim 10^{-25}$ to obtain verifiable data. Going significantly below $h \sim 10^{-23}$ involves an argument as given as follows: The following question was posed by a reviewer of a document given to Dr. Fangyu Li, and the author has reproduced Dr. Li's response below [43]

Quote:

"The most serious is that a background strain $h \sim 10^{-30}$ at 10GHz corresponds to a Ω_g (total) $\sim 10^{-3}$ which violates the baryon nuclei-synthesis epoch limit for either GWs or EMWs. Ω_g (Total) needs to be smaller than 10^{-5} otherwise the cosmological Helium/hydrogen abundance in the universe would be strongly affected....." The answer, which the author copied from Dr. Li [43], i.e. if $\nu = 10GHz, \quad h = 10^{-31}$, then $\Omega_g = 8.3 \times 10^{-7} < \Omega_{g\max}$, is an answer to this supposition

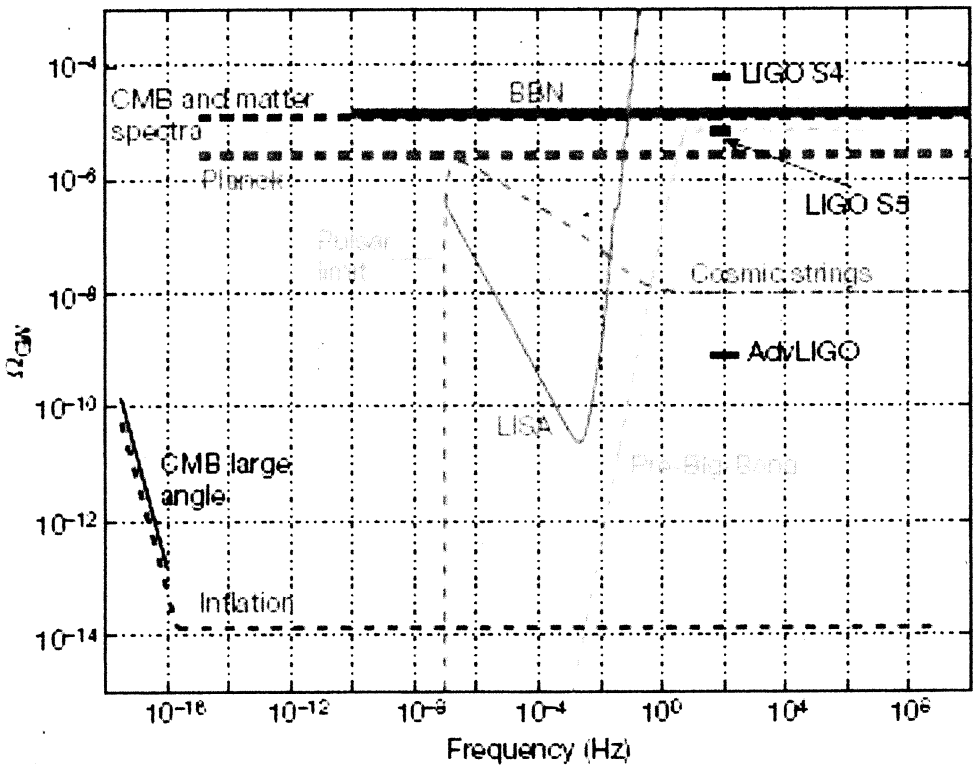

Figure3a. This figure from B.P. Abbott, et.al [1] shows the relation between Ω_g and frequency. The curve of the pre-big-bang models shows that Ω_g of the relic GWs is almost constant $\sim 6.9\times10^{-6}$ from 10^{-1}Hz to 10^{10}Hz. Ω_g of the cosmic string models is about 10^{-8} in the region 1Hz to 10^{10}Hz; its peak value region is about 10^{-7}-10^{-6}Hz. According to more accepted by the general astro physics community values, the estimate, the upper limit of Ω_g FIX THAT on relic GWs should be smaller than 10^{-5}, while recent data DATA analysis [1`] (B.P. Abbott et al, (2009)) shows the upper limit of Ω_g , as in figure FIX should be 6.9×10^{-6} FIX. By

using such parameters, Dr. Li estimates the spectrum $h(\nu_g,\tau)$ FIX and the RMS amplitude h_{rms}. The relation between Ω_g and the spectrum $h(\nu_g,\tau)$ is often expressed as [33] as, which is further amplified in [34] to read as below:

$$\Omega_g \approx \frac{\pi^2}{3}\left(\frac{\nu}{\nu_H}\right)^2 h^2(\nu,\tau), \tag{52}$$

So

$$h(\nu,\tau) \approx \frac{\sqrt{3\Omega_g}}{\pi}\frac{\nu_H}{\nu}, \tag{53}$$

Where $\nu_H = H_0 \sim 2\times10^{-18} Hz$, the present value of the Hubble frequency. From (52) and (53), we have

(a) If $\nu = 10GHz, \quad h = 10^{-30}$, then $\Omega_g = 8.3\times10^{-5}$, (54)

If $\nu = 10GHz, \quad h = 10^{-31}$, then $\Omega_g = 8.3\times10^{-7} < \Omega_{g\max}$, (55)

If $\nu = 10GHz, \quad \Omega_g = \Omega_{g\max} = 6.9\times10^{-6}$, then $h = 2.9\times10^{-31}$ (56)

(b) If $\nu = 5GHz, \quad h = 10^{-30}$ Then

$\nu = 5GHz, \quad \Omega_g = \Omega_{g\max} = 6.9\times10^{-5}$,

If $\nu = 5GHz, \quad$ h=10^{-31} then $\Omega_g = 2.1\times10^{-7} < \Omega_{g\max}$ (57)

If $\nu = 5GHz, \quad \Omega_g = \Omega_{g\max} = 6.9\times10^{-5}$, then $h = 5.7\times10^{-31}$ (58)

Such values of $\nu = 5GHz, \quad \Omega_g = \Omega_{g\max} = 6.9\times10^{-5}$, would be essential to ascertain the possibility of detection of GW from relic conditions, whereas Ω_g,

or in integral form $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c}$, as given by $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f)$.

Furthermore, one could also write the following formulation of $h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f[graviton] + n_f[neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4$ for a very narrow range of frequencies, that to first approximation, make a comparison between an integral representation of Ω_g and $h_0^2 \Omega_{gw}(f)$. Note also that Dr. Li [43]suggests, as an optimal upper frequency to investigate, $\nu_g = 2.9GHz, \Delta\nu = 3Khz$ then

$$h \approx \frac{\sqrt{3\Omega_g}}{\pi} \frac{\nu_H}{\nu_g} \approx 1.0 \times 10^{-30} , \tag{59}$$

And

$$h_{rms} = \sqrt{\langle h^2 \rangle} \approx h \left[\frac{\Delta\nu}{\nu_g} \right]^{\frac{1}{2}} \approx 1.02 \times 10^{-33} \tag{60}$$

These are upper values of the spectrum, and should be considered as preliminary. Needed in this mix of calculations would be a way to ascertain a set of input values for $n_f[graviton], n_f[neutrino]$ into $h_0^2 \Omega_{gw}(f)$. The objective is to get a set of measurements to confirm if possible the utility of using , experimentally the numerical count of

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f[graviton] + n_f[neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4 \tag{61}$$

. If there is roughly a 1-1 correspondence between gravitons and neutrios which we deem as (highly unlikely) , then

$$h_0^2\Omega_{gw}(f)\sim 3.6\cdot\left[\frac{n_f[graviton]}{10^{37}}\right]\cdot\left(\frac{\langle f\rangle}{\cdot 1kHz}\right)^4. \qquad (62)$$

counting the number of gravitons per cell space should also consider what Buoanno wrote, for Les Houches if one looks at BBN, the following upper bound should be considered:[19]

$$h_0^2\Omega_{gw}(f)\leq 4.8\times 10^{-9}\cdot(f/f_*)^2 \qquad (63)$$

Here, Buoanno [19]is using $f > f_* = 4.4\times 10^{-9} Hz$,. Using this upper bound, if one insist upon assuming, as Buoanno [19] (2006) does, that the frequency today depends upon the relation

$$f \equiv f_*\cdot[a_*/a_0] \qquad (64)$$

The problem in this is that the ratio $[a_*/a_0]$ << 1, assumes that a_0 is "today's" scale factor. In fact, using this estimate, Buoanno [19] comes up with a peak frequency value for relic early universe values of the electroweak era-generated GWgraviton production of [19]

$$f_{Peak}\cong 10^{-8}\cdot[\beta/H_*]\cdot[T_*/16GeV]\cdot[g_*/100]^{1/6}Hz \qquad (65)$$

By conventional cosmological theory, limits of g_* are at the upper limit of 100-120, at most, according to Kolb and Turner (1991) [40] . $T_*\sim 10^2 GeV$ is specified for nucleation of a bubble, as a generator of GW. Early universe

models with g_*~ 1000 or so are not in the realm of observational science, yet, according to Hector De La Vega (2009) [23] in personal communications with the author,) at the Colmo, Italy astroparticle physics school , ISAPP, Furthermore, the range of accessible frequencies as given by (65) is in sync with

$$h_0^2\Omega_{gw}(f)\sim 10^{-10} \qquad (66)$$

for peak frequencies with values of 10 MHz. The net affect of such thinking is to proclaim that all relic GW are inaccessible. If one looks at Figure , $\Omega_{GW} > 10^{-6}$ for frequencies as high as up to 10^6 Hertz, this counters what was declared by Turner and Wilzenk [71] (1990): that inflation will terminate with observable frequencies in the range of 100 or so Hertz. The problem is though, that after several years of LIGO, no one has observed such a GW signal from the early universe, from black holes, or any other source, yet. About the only way one may be able to observe a signal for GW and/or gravitons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for

$$h_0^2\Omega_{gw}(f)\cong\frac{3.6}{2}\cdot\left[\frac{n_f[graviton]+n_f[neutrino]}{10^{37}}\right]\cdot\left(\frac{\langle f\rangle}{1kHz}\right)^4 \qquad (67)$$

. And this leads to the question of how to account for a possible mass/ information content to the graviton.

10 Consequences of small graviton mass for reacceleration of the universe

Here, n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may depend upon the interaction of

gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al* , [49] which is a supposition the author is investigating for a modification of a joint KK tower of gravitons, as given by Maartens [47,48] for DM. Assume the stretching of early relic neutrinos that would lead to the KK tower of gravitons--for when $\alpha < 0$, is,

$$m_n(Graviton) = \frac{n}{L} + 10^{-65}\,\text{grams} \qquad (68)$$

. Also (68) will be the starting point used for a KK tower version of (69) below. So from Maarten's 2005 paper, [47,48]

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3}\left[\rho + \frac{\rho^2}{2\lambda}\right]\right)a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K\right] \qquad (69)$$

Maartens [47,48] also writes

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2}\cdot[p+\rho]\cdot\left[1+\frac{\rho^2}{\lambda}\right]\right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2}\right]. \qquad (70)$$

Also, if $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with equality, $\rho = -P$, for z between zero to **.5.** $a \equiv [a_0 = 1]/(1+z)$. As given by Beckwith [ ]

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1+\tilde{\kappa}^2[\rho/m]\cdot(1+z)^4\cdot(1+\rho\ /2\lambda)} \approx -1 + \frac{2}{2+\delta(z)} \qquad (71)$$

(71) assumes using (69) and. (70) $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature **K** =0 In a revision of Alves [2] , Beckwith used a higher-dimensional model of the brane world and Marsden [47,48] KK graviton towers.

The density ρ of the brane world in the Friedman equation as used by Alves *et. al.* [2] is use by Beckwith [11] for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar=1)^2}\right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2}\right) \quad (72)$$

I.e. (64) above is making a joint DM and DE model, with all of. (67) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. (71) and (68) are part of a KK graviton presentation of DM/ DE dynamics. Beckwith[8] found at $z \sim .4$, a billion years ago, that acceleration of the universe increased, as shown in Fig. 4.

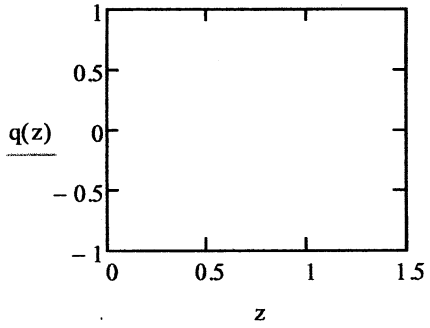

Fig. 3b: Reacceleration of the universe based on Beckwith **(note that q < 0 if z <.423)**

10 Answering Dr. Karim's questions about the Cosmological "constant" and how it may differ from Vacuum energy.

Assume that the cosmological "constant" is defined via the following equation, i.e. working with the values $\frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{Cosmological}} \sim 10^{-4} - 10^{-5}$ for varying entropy values, affecting the $\Lambda_{Cosmological}$ values, gives us a way to analyze what Dr. Karim asked, namely :[38]

At the Big Bang the only form of energy released is in the form of geometry – gravity. intense gravity field lifts vacuum fields to positive energies, So an electromagnetic vacuum of density 10^{122} kg/m3 should collapse under its own gravity. But this does not happen - that is one reason why the cosmological constant cannot be the vacuum field. Why?

How could an electromagnetic vacuum of density of 10^{122} kg/m^3 conceivably form initially in the first place ? And what are the procedures which could determine if there is, or is not, a connection between $\Lambda_{Cosmological}$ and vacuum energy ?

The earliest treated version of a changing cosmological constant the author referenced was in having $\Lambda_{Cosmological} \sim c_1 T^{\beta}$ [60], with T in this case a measure of background temperature. If one is trying to relate $\Lambda_{Cosmological}$ strictly to a vacuum energy state, initially, the following integral equations of motion become relevant to the inquiry. Namely look at a change of variables, with $\tau = H_0 t$, with H_0 the present Hubble parameter value, and with $a = a_0 q(\tau)$ so that "particles" moving in a particular cosmology operate under, as given by Padmanabhan which is [58,59] , for 'particles' in a trajectory semi classically defined by

$$\frac{1}{2}\cdot\left[\frac{dq}{d\tau}\right]^2 + V(q) = 0 \qquad (73)$$

This assuming that the potential as given, namely $V(q)$ have the following contents in initial universe conditions

$$V(q) \cong -(1/2) \cdot \frac{\Omega_{radiation}}{q^2} \qquad (74)$$

The expression for $\Omega_{radiation}$could be related to an initially huge cosmological constant, which would die out in time, as stating succinctly that up to a degree one can write when $\rho_{critial} \equiv 3M_P^2 H_{initial}^2$, i.ė. signifying a roughly spatially flat cosmological constant regime in the vicinity of the inflationary era of cosmology.

$$\Omega \sim \Omega_{radiation} \equiv \frac{\rho_{radiation}}{\rho_{critial}} \approx \frac{\rho_{radiation}}{3M_P^2 H_{initial}^{\ 2}} \sim \frac{\left[\frac{\pi^2}{30} \cdot g_* T^4\right]}{3M_P^2 H_{initial}^{\ 2}} \qquad (75)$$

Recall that Park,[60] in 2003, wrote an effective early cosmological "constant" parameter as $\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}}$

So that up to a point, one could write, for the early temperature dependence, having

$$\Omega \sim \frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot \left[\Lambda_{Max} / c_2\right]^{4/\tilde{\beta}}\right]}{3M_P^2 H_{initial}^{\ 2}} \sim \frac{\rho_{early-universe}}{3M_P^2 H_{initial}^{\ 2}} \leq 1 \qquad (76)$$

The denominator in place only makes sense if there is 'flat space' and the term for $\rho_{critial} \equiv 3M_P^2 H_{initial}^2$ cannot be defined for when curved space conditions take hold.

Case I, $\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}}$ is a vacuum energy term. The problem is, what to do in the case that the temperature reaches $T \leq 1.416 \times 0^{32} K \sim 10^{19} GeV$ as opposed to the Planck Mass term of setting the value of $M_P \sim 1.2 \times 10^{19} GeV/c^2$. Then the following ratio as can be rounded off leaves

$$\Omega \sim \frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot \left[1.416 \times 10^{19} GeV\right]^4\right]}{3 \cdot \left[1.2 \times 10^{19} GeV\right]^2 H_{initial}^{\ 2}} \sim \frac{\frac{\pi^2}{30} \cdot g_* \cdot c^4}{3} \cdot \frac{\left[10^{19} GeV\right]^2}{H_{initial}^2} \geq 1 \quad (77)$$

I.e. especially if the degrees of freedom rises above $g_* \geq 120$. Note that $g_* \approx 120$ at T ~ 100 KeV Unless the term for $H_{initial}$ were absolutely enormous, and if $g_* \approx 1000$, then $\Omega \geq 1$ could happen, which would be physically meaningless. The other situation is that there could be situations for which g_* would be undefined, especially if $T \leq 1.416 \times 0^{32} K \sim 10^{19} GeV$ were close to an equality. We state here unequivocally that (77) above is likely important, and that this has serious experimental import. i.e. to a study of fundamental processes inherent to the task of making gravitational astronomy a falsifiable, experimentally based science.

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EXTERIOR AND INTERIOR DYNAMIC ISO-SPHERE HOLOGRAPHIC RINGS WITH AN INVERSE ISO-DUALITY

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Abstract

In this preliminary work, we use a dynamic iso-unit function to iso-topically lift the "static" Inopin holographic ring (IHR) of the unit sphere to an interconnected pair of "dynamic iso-sphere IHRs" (iso-DIHR), where the IHR is simultaneously iso-dual to both a magnified "exterior iso-DIHR" and de-magnified "interior iso-DIHR". For both the continuously-varying and discretely-varying cases, we define the dynamic iso-amplitude-radius of one iso-DIHR as being equivalent to the dynamic iso-amplitude-curvature of its counterpart, and conversely. These initial results support the hypothesis that a new IHR-based mode of iso-geometry and iso-topology may be in order, which is significant because the interior and exterior zones delineated by the IHR are fundamentally "iso-dual inverses" and may be inferred from one another.

Keywords: Santilli iso-mathematics; Inopin holographic ring; Iso-geometry; Iso-topology; Dynamic iso-sphere; Iso-duality.

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To the memory of my good friend Gavin Koester-Backstrom.

1 Introduction

In a forward attempt to establish order in chaos, A.E. Inopin introduced the dual space-time IHR topology in a proof of quark confinement [1], which received a preliminary topological upgrade in the triplex generalization of [2]. In Euclidean complex space, Inopin's dual 3D space-time IHR topology comprises a 1-sphere IHR "time zone" that delineates two spatial 2-branes, whereas in Euclidean triplex space, Inopin's dual 4D space-time IHR topology generalizes the 1-sphere IHR to a 2-sphere IHR that delineates two spatial 3-branes [1, 2]. In other words, the brane states can be inferred from the IHR states and vice-versa because the IHR acquires Berry phase transitions for (spontaneous gauge symmetry breaking) topological deformation order parameters and is simultaneously dual to both branes [1, 2].

Recently, R.M. Santilli's iso-mathematics [3, 4, 5, 6, 7] was applied to the dual 4D space-time IHR topology (with the 2-sphere IHR) [1, 2, 8] to initiate the *iso-dual* 4D space-time IHR topology (with the iso-2-sphere IHR) [9]. Subsequently, the new class of *dynamic iso-spaces* was constructed [10]; a dynamic iso-space is an iso-space that is characterized by constant *change* [10]. More specifically, a dynamic iso-space is built with a dynamic isotopic lifting that arises due to a dynamic iso-unit function that varies over time [10]. Santilli's discovery of iso-mathematics gave way to these dynamic constructs because he proved that his iso-unit can be, among many things, a function [3, 4, 5, 6, 7]. Therefore, in this paper, we engage this cutting-edge notion of dynamic iso-spaces [10] with the emerging iso-dual 4D space-time IHR topology [9] to define an interconnected pair of iso-DIHRs, where the exterior iso-DIHR is iso-dual to an interior iso-DIHR.

We launch our investigation with Section 2, where we augment the iso-sphere IHR (iso-IHR) [9] by initiating definitions for the *exterior iso-IHR* and the *interior iso-IHR*—for this, we propose a fundamental and critical *inverse iso-duality* between the exterior and interior iso-IHRs that are both locally iso-morphic to the original IHR. Next, in Section 3, we deploy the dynamic iso-topic lifting of [10] to upgrade the initial results of Section 2, where we mobilize general, continuous, and discrete definitions

for the exterior iso-DIHR and interior iso-DIHR that preserve the original iso-morphism. Finally, we conclude our exploration with the recapitulation of results and future outlook of Section 4.

2 Inverse iso-duality between the exterior and interior iso-sphere IHRs

Here, we discuss and extend the iso-IHR [9] by proposing an inverse iso-duality between the exterior and interior branes.

Following [1, 2], let T_r^1 be the *unit 1-sphere IHR* of amplitude-radius $r = 1$ and amplitude-curvature $\kappa = \frac{1}{r}$ that is iso-metrically embedded in the complex space S^2, such that eq. (13) of [2] identifies

$$T_r^1 = \{\vec{s} \in S^2 : |\vec{s}| = r\}, \tag{1}$$

where $T_r^1 \subset S^2$ is the multiplicative group of all non-zero complex coordinate-vectors of normalized amplitude-radius r. Next, let T_r^2 be a unit 2-sphere IHR that is iso-metrically embedded in the triplex space S^3, such that eq. (40) of [2] identifies

$$T_r^2 = \{\vec{s} \in S^3 : |\vec{s}| = r\}, \tag{2}$$

where $T_r^2 \subset S^3$ is the multiplicative group of all non-zero triplex coordinate-vectors of normalized amplitude-radius r; T_r^1 is the great circle of T_r^2 so both *non-linear* structures share the same amplitude-radius r with the constant characterizing curvature of κ, where $S^2 \subset S^3$ and $T_r^1 = T_r^2 \cap S^2$ [2]. In this IHR-based topology [1, 2], eqs. (14) and (40) in [2] demonstrate that the micro 2-brane sub-space $S_-^2 \subset S^2$ and the micro 3-brane sub-space $S_-^3 \subset S^3$ correspond to *interior* dynamical systems, while the macro 2-brane sub-space $S_+^2 \subset S^2$ and the micro 3-brane sub-space $S_+^3 \subset S^3$ correspond to *exterior* dynamical systems, where T_r^1 delineates S_-^2 and S_+^2, and T_r^2 delineates S_-^3 and S_+^3.

Now, following Santilli's iso-number methodology [3, 4, 5, 6, 7] and the iso-IHR definition [9], we select some positive-definite iso-unit $\hat{r}_+ > r$ with the corresponding positive-definite inverse $\hat{r}_- = \frac{1}{\hat{r}_+} < r$ to establish the array of *exterior* iso-topic liftings

$$\begin{array}{rrcl} f(\hat{r}_+): & T_r^n & \to & T_{\hat{r}_+}^n \\ f^{-1}(\hat{r}_+): & T_{\hat{r}_+}^n & \to & T_r^n \end{array}, \quad n \in \{1, 2\}, \tag{3}$$

for the magnified

1. *exterior iso-1-sphere IHR (iso-1-IHR)* $T^1_{\hat{r}_+}$ and

2. *exterior iso-2-sphere IHR (iso-2-IHR)* $T^2_{\hat{r}_+}$.

In this case of eq. (3), a given $T^n_{\hat{r}_+}$ is "outside" T^n_r because $\hat{r}_+ > r$. Thus, $\hat{r}_+$ is termed the *exterior iso-unit*, which serves as the *exterior iso-amplitude-radius* for both $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, while $\hat{r}_-$ serves as the *exterior iso-amplitude-curvature* for both $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, such that $T^1_{\hat{r}_+} \subset T^2_{\hat{r}_+}$ is the great circle of $T^2_{\hat{r}_+}$.

So a question comes to mind: how might the exterior iso-amplitude-curvature $\hat{r}_-$ fit into the structure and function of the said iso-IHR topology? Our *hypothesis* is that the selected iso-unit relation $\hat{r}_+ = \frac{1}{\hat{r}_-} > r$ identifies a critical and fundamental iso-duality for IHR-based topological implementations in terms of spherical radii and curvature. In the iso-IHR topology introduction of [9], we recall that the iso-amplitude-curvature property was only mentioned in a brief context due to the limited scope of that analysis. Therefore, in this section, we wish to further probe the applicability of the iso-amplitude-curvature by deploying it to define an additional topological iso-structure. Hence, in addition to being the exterior iso-amplitude-curvature of $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, we furthermore define $\hat{r}_-$ as the *interior iso-amplitude-radius* and *interior iso-unit* of two *new* iso-IHRs, namely the de-magnified

1. *interior iso-1-IHR* $T^1_{\hat{r}_-}$ and

2. *interior iso-2-IHR* $T^2_{\hat{r}_-}$,

with the corresponding array of *interior* iso-topic liftings

$$\begin{array}{rrcl} f(\hat{r}_-): & T^n_r & \rightarrow & T^n_{\hat{r}_-} \\ f^{-1}(\hat{r}_-): & T^n_{\hat{r}_-} & \rightarrow & T^n_r \end{array}, \quad n \in \{1, 2\}. \tag{4}$$

In this case of eq. (4), a given $T^n_{\hat{r}_-}$ is "inside" T^n_r because $\hat{r}_- < r$. Hence, upon recalling the relation $\hat{r}_- = \frac{1}{\hat{r}_+}$, we realize that $\hat{r}_+$ is also the *interior iso-amplitude-curvature* of both $T^1_{\hat{r}_-}$ and $T^2_{\hat{r}_-}$! Thus, in terms of iso-

amplitude-radius and iso-amplitude-curvature, we've identified a *fundamental iso-duality* between $T^n_{\hat{r}_+}$ and $T^n_{\hat{r}_-}$ written as

$$T^n_{\hat{r}_-} \leftarrow T^n_r \rightarrow T^n_{\hat{r}_+} \tag{5}$$

because the iso-amplitude-radii and iso-amplitude-curvatures are interdependent with respect to T^n_r. Therefore, in addition to the lemmas of [9], the results of eqs. (3–5) indicate the trichôtomy:

1. ($\hat{r}_- < 1$): interior iso-amplitude-radius of $T^n_{\hat{r}_-}$, exterior iso-amplitude-curvature of $T^n_{\hat{r}_+}$, interior iso-multiplicative iso-unit of $T^n_{\hat{r}_-}$, exterior iso-multiplicative iso-unit inverse of $T^n_{\hat{r}_+}$;

2. ($r = 1$): amplitude-radius, amplitude-curvature, multiplicative unit; and

3. ($\hat{r}_+ > 1$): exterior iso-amplitude-radius of $T^n_{\hat{r}_+}$, interior iso-amplitude-curvature of $T^n_{\hat{r}_-}$, exterior iso-multiplicative iso-unit of $T^n_{\hat{r}_+}$, interior iso-multiplicative iso-unit inverse of $T^n_{\hat{r}_-}$.

Therefore, we have establish the following:

Lemma 1. *An n-sphere IHR T^n_r of amplitude-radius (and unit) $r = 1$ that is iso-topically lifted via $T^n_r \rightarrow T^n_{\hat{r}_+}$ to the exterior iso-n-IHR $T^n_{\hat{r}_+}$ of exterior iso-amplitude-radius (and exterior iso-multiplicative iso-unit) $\hat{r}_+ > r$ can be simultaneously lifted via $T^n_r \rightarrow T^n_{\hat{r}_-}$ to the interior iso-n-IHR $T^n_{\hat{r}_-}$ of interior iso-amplitude-radius (and interior iso-multiplicative iso-unit) $\hat{r}_- < r$ if $\hat{r}_- = \frac{1}{\hat{r}_+}$, where $\hat{r}_+$ is the interior iso-amplitude-curvature of $T^n_{\hat{r}_-}$ and $\hat{r}_-$ is the exterior iso-amplitude-curvature of $T^n_{\hat{r}_+}$, such that $T^n_{\hat{r}_+}$ and $T^n_{\hat{r}_-}$ are iso-dual inverses and locally iso-morphic to T^n_r.*

At this point, we've discussed and extended the iso-IHR of [9] to include the exterior and interior iso-IHRs of eqs. (3–4) and Lemma 1. See Figure 1 for a depiction of this scenario.

3 Dynamic iso-sphere IHR

Here, we apply the dynamic iso-topic lifting of [10] to the iso-IHR results of Section 2, where we'll introduce the general definitions for the exterior and interior iso-DIHRs in Section 3.1. Subsequently, in Section 3.2, we'll push beyond the general form to construct the continuous and discrete cases.

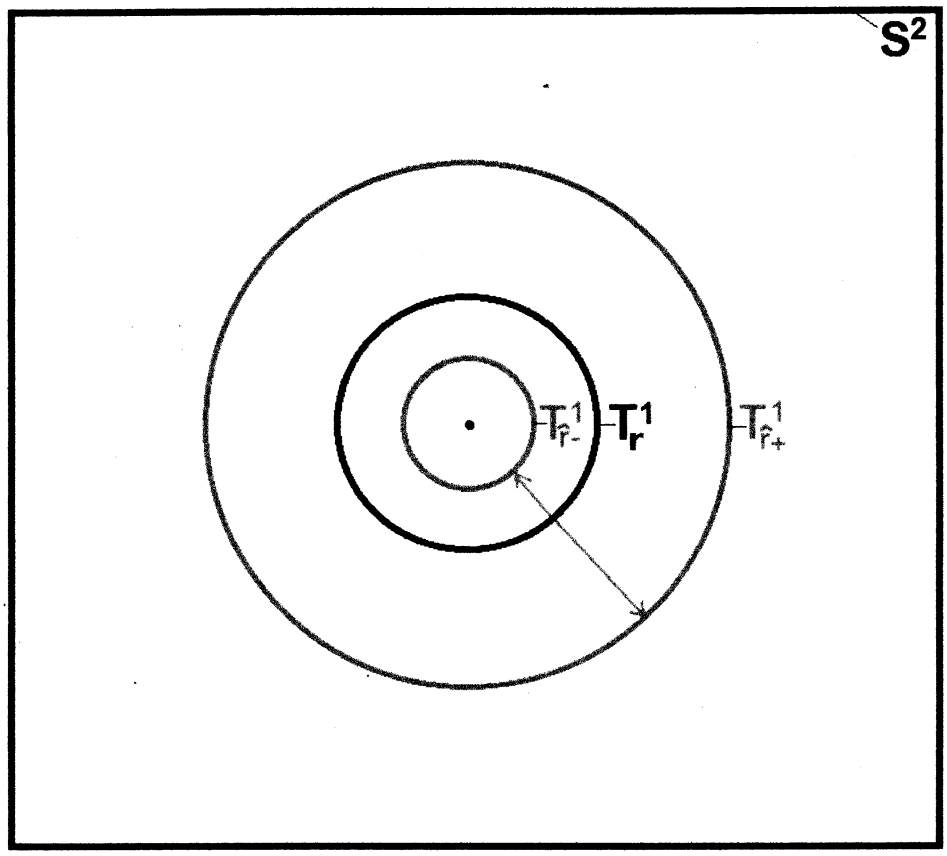

Fig. 1: The iso-1-IHR T^1_r is iso-topically lifted to both the exterior iso-1-IHR $T^1_{\hat{r}_+}$ and the interior iso-1-IHR $T^1_{\hat{r}_-}$ simultaneously, where $T^1_{\hat{r}_+}$ and $T^1_{\hat{r}_-}$ are iso-dual.

3.1 General

Thus, following the dynamic methodology of [10], we define the positive-definite dynamic iso-unit function as

$$\hat{r}_+ \equiv \hat{\delta}_+(t) > r \tag{6}$$

with its corresponding positive-definite inverse

$$\hat{r}_- \equiv \frac{1}{\hat{\delta}_+(t)} \equiv \hat{\delta}_-(t) < r, \tag{7}$$

where $\hat{\delta}_+(t)$ increases and $\hat{\delta}_-(t)$ decreases simultaneously as the parameter t varies as $t \to \infty$, for the *general* form. Hence, eq. (3) can be rewritten to establish the *exterior dynamic iso-topic lifting* form

$$\begin{array}{rrcl} f(\hat{\delta}_+(t)): & T_r^n & \to & T_{\hat{\delta}_+(t)}^n \\ f^{-1}(\hat{\delta}_+(t)): & T_{\hat{\delta}_+(t)}^n & \to & T_r^n \end{array}, \quad n \in \{1,2\}, \tag{8}$$

to define the

1. *exterior iso-1-DIHR* $T_{\hat{\delta}_+(t)}^1$ and
2. *exterior iso-2-DIHR* $T_{\hat{\delta}_+(t)}^2$.

Similarly, eq. (4) can be rewritten to express the *interior dynamic iso-topic lifting* form

$$\begin{array}{rrcl} f(\hat{\delta}_-(t)): & T_r^n & \to & T_{\hat{\delta}_-(t)}^n \\ f^{-1}(\hat{\delta}_-(t)): & T_{\hat{\delta}_-(t)}^n & \to & T_r^n \end{array}, \quad n \in \{1,2\}, \tag{9}$$

to define the

1. *interior iso-1-DIHR* $T_{\hat{\delta}_-(t)}^1$ and
2. *interior iso-2-DIHR* $T_{\hat{\delta}_-(t)}^2$.

Therefore, the implications and results of eqs. (6–9) authorize us to establish the following:

Lemma 2. *An n-sphere IHR T^n_r of amplitude-radius (and unit) $r = 1$ that is dynamically iso-topically lifted via $T^n_r \to T^n_{\hat{\delta}_+(t)}$ to the exterior iso-n-DIHR $T^n_{\hat{\delta}_+(t)}$ of exterior dynamic iso-amplitude-radius (and exterior dynamic iso-unit) $\hat{\delta}_+(t) > r$ can be simultaneously lifted via $T^n \to T^n_{\hat{\delta}_-(t)}$ to the interior iso-n-DIHR $T^n_{\hat{\delta}_-(t)}$ of interior dynamic iso-amplitude-radius (and interior dynamic iso-unit) $\hat{\delta}_-(t) < r$ if $\hat{\delta}_-(t) = \frac{1}{\hat{\delta}_+(t)}$ as the parameter t varies, where $\hat{\delta}_+(t)$ is the interior dynamic iso-amplitude-curvature of $T^n_{\hat{\delta}_-(t)}$ and $\hat{\delta}_-(t)$ is the exterior dynamic iso-amplitude-curvature of $T^n_{\hat{\delta}_+(t)}$, such that $T^n_{\hat{\delta}_+(t)}$ and $T^n_{\hat{\delta}_-(t)}$ are dynamically, inversely iso-dual and locally iso-morphic to T^n_r.*

At this point, we've successfully applied the *general* dynamic iso-topic lifting definitions of [10] to the iso-IHR results of Section 2 by introducing the definitions for the exterior and interior iso-DIHRs in Section 3.1, where the resulting constructions of eqs. (6–9) are characterized by Lemma 2. See Figure 2 for a depiction of this scenario.

3.2 Continuous and discrete

Next, we combine the continuous and discrete dynamic iso-space definitions of [10] with the general iso-DIHR definitions of Section 3.1 to assemble the continuous and discrete iso-DIHR implementations.

First, we will show that $T^n_{\hat{\delta}_-(t)}$ and $T^n_{\hat{\delta}_+(t)}$ can be defined as *continuous iso-n-DIHRs* if the dynamic iso-unit functions $\hat{\delta}_{+_c}(t)$ and $\hat{\delta}_{-_c}(t)$ are both continuous as their parameter t varies, where insert the additional subscript label c to denote the "continuous" case. Hence, for example, let t be the continuously varying parameter for the *continuous exterior and interior dynamic iso-unit* functions

$$\begin{aligned} \hat{r}_+ &\equiv \hat{\delta}_{+_c}(t) \in \mathbb{R}_c \\ \hat{r}_- &\equiv \hat{\delta}_{-_c}(t) \equiv \tfrac{1}{\hat{\delta}_{+_c}(t)} \in \mathbb{R}_c \end{aligned}, \quad 0 < \hat{\delta}_{-_c}(t) < r < \hat{\delta}_{+_c}(t) < \infty, \ t \to \infty, \tag{10}$$

such that $\mathbb{R}_c$ is a positive-definite continuous set (i.e. the positive real numbers), to consequently define the

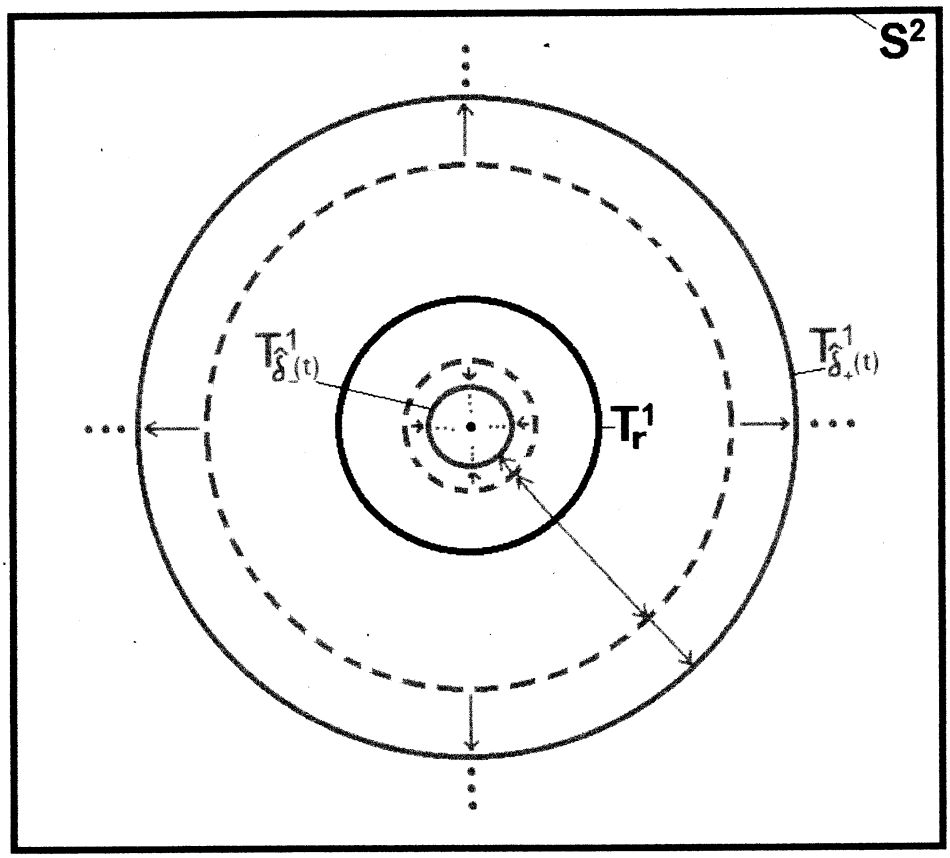

Fig. 2: The iso-1-IHR T^1_r is dynamically iso-topically lifted to both the exterior iso-1-DIHR $T^1_{\hat{\delta}_+(t)}$ and the interior iso-1-DIHR $T^1_{\hat{\delta}_-(t)}$ simultaneously as the parameter t varies as $t \to \infty$, where $T^1_{\hat{\delta}_+(t)}$ and $T^1_{\hat{\delta}_-(t)}$ are iso-dual inverses.

1. *continuous exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+_c}(t)}$ and
2. *continuous interior iso-n-DIHR* $T^n_{\hat{\delta}_{-_c}(t)}$,

where we rewrite eqs. (8–9) in the *continuous exterior and interior dynamic iso-topic lifting* form

$$\begin{array}{rrcl} f(\hat{\delta}_{+_c}(t)): & T^n_r & \rightarrow & T^n_{\hat{\delta}_{+_c}(t)} \\ f^{-1}(\hat{\delta}_{+_c}(t)): & T^n_{\hat{\delta}_{+_c}(t)} & \rightarrow & T^n_r \end{array}, \quad n \in \{1,2\}, \tag{11}$$

and

$$\begin{array}{rrcl} f(\hat{\delta}_{-_c}(t)): & T^n_r & \rightarrow & T^n_{\hat{\delta}_{-_c}(t)} \\ f^{-1}(\hat{\delta}_{-_c}(t)): & T^n_{\hat{\delta}_{-_c}(t)} & \rightarrow & T^n_r, \end{array}, \quad n \in \{1,2\}, \tag{12}$$

respectively. In eqs. (11–12), T^n_r remains locally iso-morphic to both $T^n_{\hat{\delta}_{-_c}(t)}$ and $T^n_{\hat{\delta}_{+_c}(t)}$ as t continuously varies. Thus, the results of eqs. (10–12) permit us to identify the following:

Lemma 3. *An exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+_c}(t)}$ *is a continuous exterior iso-n-DIHR if the exterior dynamic iso-unit function* $\hat{\delta}_{+_c}(t)$ *is continuous as its parameter* t *varies.*

Lemma 4. *An interior iso-n-DIHR* $T^n_{\hat{\delta}_{-_c}(t)}$ *is a continuous interior iso-n-DIHR if the interior dynamic iso-unit function* $\hat{\delta}_{-_c}(t)$ *is continuous as its parameter* t *varies.*

Second, we will show that $T^n_{\hat{\delta}_{-}(t)}$ and $T^n_{\hat{\delta}_{+}(t)}$ can also be defined as *discrete iso-n-DIHRs* if the dynamic iso-unit functions $\hat{\delta}_{+_d}(t)$ and $\hat{\delta}_{-_d}(t)$ are both discrete as their parameter t varies, where let d denote the "discrete" case. Hence, for example, let t be the discretely varying parameter for the *discrete exterior and interior dynamic iso-unit* functions

$$\begin{array}{rcl} \hat{r}_+ & \equiv & \hat{\delta}_{+_d}(t) \in \mathbb{R}_d \\ \hat{r}_- & \equiv & \hat{\delta}_{-_d}(t) \equiv \frac{1}{\hat{\delta}_{+_d}(t)} \in \mathbb{R}_d \end{array}, \; 0 < \hat{\delta}_{-_d}(t) < r < \hat{\delta}_{+_d}(t) < \infty, \; t \rightarrow \infty, \tag{13}$$

such that $\mathbb{R}_d$ is a positive-definite discrete set (i.e. positive Fibonacci numbers), to consequently define the

1. *discrete exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+_d}(t)}$ and
2. *discrete interior iso-n-DIHR* $T^n_{\hat{\delta}_{-_d}(t)}$,

where we rewrite eqs. (8–9) in the *discrete exterior and interior dynamic iso-topic lifting* form

$$\begin{array}{rrcl} f(\hat{\delta}_{+_d}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{+_d}(t)} \\ f^{-1}(\hat{\delta}_{+_d}(t)): & T^n_{\hat{\delta}_{+_d}(t)} & \to & T^n_r \end{array}, \quad n \in \{1,2\}, \tag{14}$$

and

$$\begin{array}{rrcl} f(\hat{\delta}_{-_d}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{-_d}(t)} \\ f^{-1}(\hat{\delta}_{-_d}(t)): & T^n_{\hat{\delta}_{-_d}(t)} & \to & T^n_r, \end{array}, \quad n \in \{1,2\}, \tag{15}$$

respectively. In eqs. (14–15), $T)r^n$ remains locally iso-morphic to both $T^n_{\hat{\delta}_{-_d}(t)}$ and $T^n_{\hat{\delta}_{+_d}(t)}$ as t discretely varies. Thus, the results of eqs. (13–15) enable us to identify the following:

Lemma 5. *An exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+_d}(t)}$ *is a discrete exterior iso-n-DIHR if the exterior dynamic iso-unit function* $\hat{\delta}_{+_d}(t)$ *is discrete as its parameter* t *varies.*

Lemma 6. *An interior iso-n-DIHR* $T^n_{\hat{\delta}_{-_d}(t)}$ *is a discrete interior iso-n-DIHR if the interior dynamic iso-unit function* $\hat{\delta}_{-_d}(t)$ *is discrete as its parameter* t *varies.*

At this point, we've successfully combined the continuous and discrete dynamic iso-space definitions of [10] with the general iso-DIHR definitions of Section 3.1 to assemble the continuous and discrete iso-DIHR implementations, where the resulting constructions of eqs. (10–15) are characterized by Lemmas 3–6.

4 Conclusion

The results of this work include original definitions and lemmas for continuous and discrete iso-DIHRs. Through this process, we proposed an "inverse iso-duality" that fundamentally relates the exterior iso-DIHRs to the interior iso-DIHRs, which are simultaneously, locally iso-morphic to the original IHR. This emerging array of iso-DIHRs is significant because it extends the Santilli's pioneering work [3, 4, 5, 6, 7] to new realms of exploration with potential (near future) application to the disciplines of science, technology, and engineering.

Thus, there is still much work to do, as we must continue to relentlessly scrutinize, challenge, and upgrade this emerging framework via the Scientific Method. In particular, we suggest that in order to test the validity of our results and advance the general capability and applicability of these dynamic systems to subsequent levels, a thorough and rigorous iso-mathematical investigation should be conducted along this research trajectory. For this, we must prove the said lemmas and expand the framework by instantiating additional pertinent IHR families of dynamic iso-spheres, and furthermore the dynamic geno-spheres, dynamic hyper-spheres, and dynamic iso-dual-spheres.

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THE GALACTIC BLACK HOLE

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Abstract

Many galaxies have a concentration of mass at their center. In what follows, the mass is attributed to a neutral gas of electrons and positrons. It is found that electron degeneracy pressure supports the smaller masses against gravity. The larger masses are supported by ideal gas and radiation pressure. Physical properties are calculated for the range 450 to 45 billion solar masses.

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1 Introduction

It is now well-established that many galaxies harbor a concentration of mass at their center. It is thought that similar concentrations exist in the nuclei of active galaxies and quasars. The question arises as to the physical nature of these masses. In the following model, electrons and positrons form a neutral gas which is confined by gravitation. Such objects would have formed during the Big Bang, when positrons were plentiful.

The paper begins with a calculation of electron degeneracy pressure. When applied to the electron-positron gas, it is found that the radius decreases with increasing mass, as is the case with white dwarf stars [1]. This process continues until the Schwarzschild radius $R_s = 2GM/c^2$ is reached. This occurs at about $10^6\,\mathrm{M}_\odot$. It is postulated that the radius can be no smaller than R_s. With further additions of mass, R_s increases. In these supermassive objects, degeneracy pressure gives way to ideal gas and radiation pressure.

2 The electron-positron gas

Under the assumption of spherical symmetry, the gravitational field equation and the equation of hydrostatic equilibrium are given by

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = 4\pi G\rho \tag{1}$$

and

$$\frac{dP}{dr} = -\rho\frac{d\psi}{dr} \tag{2}$$

With the additional assumption of constant density, they yield the pressure formula

$$P = \frac{3GM^2}{8\pi R^4}\left(1 - \frac{r^2}{R^2}\right) \tag{3}$$

At $r = 0$, the pressure is

$$P_c = \frac{3GM^2}{8\pi R^4} \tag{4}$$

This equation will be used to determine the physical conditions at the center.

2.1 Electron degeneracy pressure

For a completely degenerate ($T = 0$) electron gas, the Fermi energy is

$$\epsilon_F = (3\pi^2)^{2/3}\frac{\hbar^2}{2m}\left(\frac{N^-}{V}\right)^{2/3} \tag{5}$$

where N^- is the number of electrons [2, 3]. The condition of neutrality is

$$N^- = N^+ = \frac{1}{2}N \tag{6}$$

where N is the number of leptons. Therefore, the Fermi energy for both electrons and positrons is

$$\epsilon_F = \left(\frac{3\pi^2}{2}\right)^{2/3}\frac{\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3} \tag{7}$$

The pressure of each gas is

$$P^- = P^+ = \left(\frac{3\pi^2}{2}\right)^{2/3}\frac{\hbar^2}{10m}\left(\frac{N}{V}\right)^{5/3} \tag{8}$$

so that the total degeneracy pressure is

$$P_F = \left(\frac{3\pi^2}{2}\right)^{2/3}\frac{\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3} \tag{9}$$

Substitution into (4) gives

$$\left(\frac{3\pi^2}{2}\right)^{2/3}\frac{\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3} = \frac{3GM^2}{8\pi R^4} \tag{10}$$

Set $M = Nm$ and $V = 4\pi R^3/3$ then rearrange to find

$$R = \frac{2}{5}\left(\frac{9\pi}{8}\right)^{2/3}\frac{\hbar^2}{Gm^3}N^{-1/3} \tag{11}$$

Clearly, the radius decreases as N increases. Solutions for these intermediate masses are as follows:

Table 1: Intermediate mass					
N	n (cm^{-3})	M ($M_\odot$)	$R > R_s$ (cm)	P_c (Pa)	ϵ_F (eV)
10^{63}	$3(10^{22})$	450	$2(10^{13})$	$4(10^{9})$	2.14
10^{64}	$3(10^{24})$	$4.5(10^{3})$	$9(10^{12})$	$8(10^{12})$	45.8
10^{65}	$3(10^{26})$	$4.5(10^{4})$	$4(10^{12})$	$2(10^{16})$	990
10^{66}	$3(10^{28})$	$4.5(10^{5})$	$2(10^{12})$	$4(10^{19})$	$2.1(10^{4})$
$8(10^{66})$	$2(10^{30})$	$3.5(10^{6})$	(10^{12})	$3(10^{22})$	$3.2(10^{5})$

In the final line, $R = R_s$.

2.2 Ideal gas and radiation pressure

In objects with $R = R_s$, the ordered kinetic energy of the quantum gas is replaced by the disordered thermal energy of an ideal gas. The electromagnetic radiation is trapped by gravity, and the temperature rises dramatically. With the ideal gas law and the Stefan-Boltzmann law for radiation pressure, equation (4) becomes

$$P_g + P_r = \frac{N}{V}kT + \frac{\pi^2}{45}\frac{(kT)^4}{(\hbar c)^3} = \frac{3GM^2}{8\pi R^4} \tag{12}$$

Set $R = 2GM/c^2$ and rearrange to obtain

$$N^2\left(\frac{kT}{mc^2}\right)^4 = \frac{135}{128\pi^3}\left(\frac{\hbar c}{Gm^2}\right)^3\left(1 - \frac{4kT}{mc^2}\right) \tag{13}$$

Solutions are as follows:

Table 2: Supermassive					
N	n (cm^{-3})	M ($M_\odot$)	$R = R_s$ (cm)	P_c (Pa)	kT (eV)
10^{67}	$9.7(10^{29})$	$4.5(10^{6})$	$1.35(10^{12})$	$2(10^{22})$	$1.2(10^{5})$
10^{68}	$9.7(10^{27})$	$4.5(10^{7})$	$1.35(10^{13})$	$2(10^{20})$	$6.7(10^{4})$
10^{69}	$9.7(10^{25})$	$4.5(10^{8})$	$1.35(10^{14})$	$2(10^{18})$	$2.4(10^{4})$
10^{70}	$9.7(10^{23})$	$4.5(10^{9})$	$1.35(10^{15})$	$2(10^{16})$	$8.0(10^{3})$
10^{71}	$9.7(10^{21})$	$4.5(10^{10})$	$1.35(10^{16})$	$2(10^{14})$	$2.5(10^{3})$

With increasing mass, the pressure, density, and temperature all diminish. The ratio of ideal gas to radiation pressure is given by

$$\frac{P_g}{P_r} = \frac{45}{\pi^2}\left(\frac{\hbar c}{kT}\right)^3 \frac{N}{V} \tag{14}$$

Because of the high density, ideal gas pressure dominates near the transition.

3 Concluding remarks

The gravitational force at the Schwarzschild surface scales as $1/R$. It becomes quite feeble in the largest masses. An ambient disturbance could remove matter from the surface. It would then become more compact and stable. In contrast, the lightest intermediate masses have a vanishingly small Fermi energy. Ambient conditions could disrupt the equilibrium, to the point that the mass disappears altogether. This may have occurred in those galaxies which have little or no central mass.

In the transition region, 10^6 to $10^7\,\mathrm{M}_\odot$, the temperature appears to change abruptly. However, the observed masses seem to form a continuum. A more detailed treatment is needed, involving a mixture of partially degenerate, ideal gas, and radiative contributions.

This paper ignores many issues, such as the variation of density with pressure, the attenuation of waves below the plasma frequency, and the large gravitational redshift as radiation moves upward from the center. Nevertheless, the work demonstrates the feasibility of the electron-positron gas, showing that it is able to account for the galactic central mass over a very wide range of values.

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